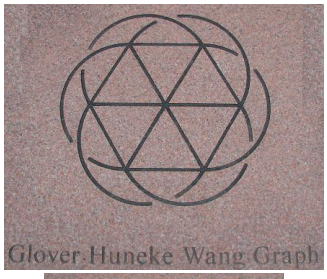
Knots and Graphs

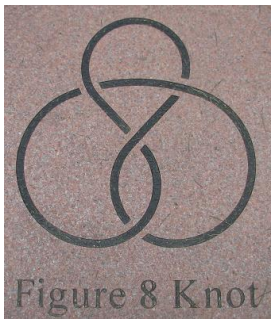
Undergraduate Research Working Group [Summer 2013]

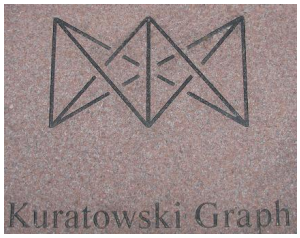
Instructor: ***Sergei Chmutov***

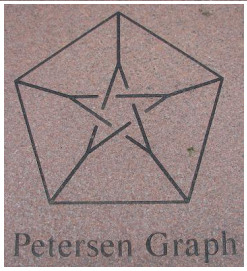
ANNOUNCEMENT

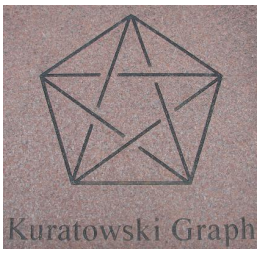
Everyone entering Math Tower put his feet on Knots and Graphs.
We are going to put our hands on them.



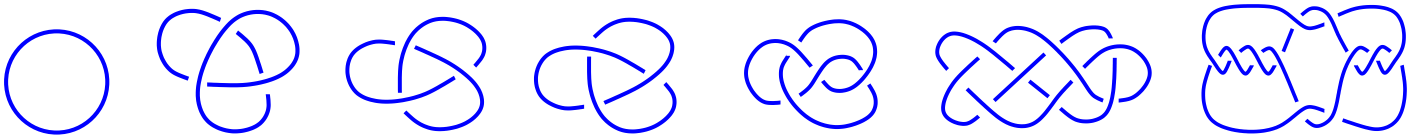




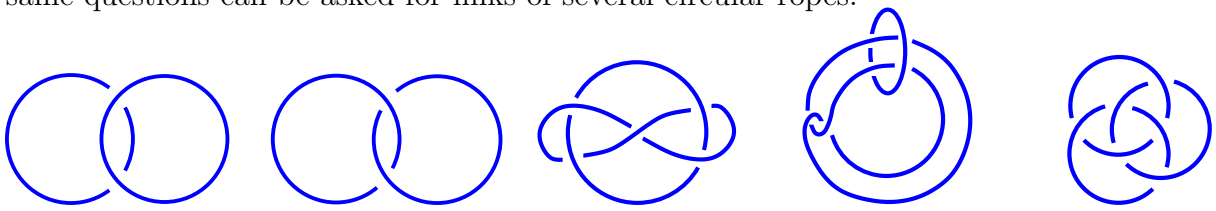




Imagine a rope tied somehow and have ends glued together afterward, forming a circular tied rope. Having two such ropes, can you move one of them around to get the other one? In particular, can you untie it and get a perfect circle? Take a look at the ropes below and think which of them you can untie, which you can not, for which two of them you can move one around and get the other.

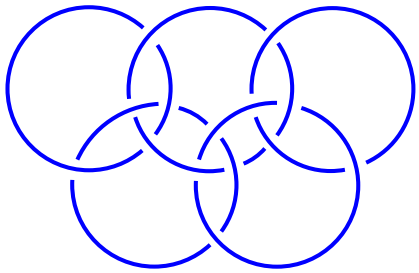


The same questions can be asked for links of several circular ropes:



The last link is known as "Borromean rings"'. It has a nice property that if any component is removed then the other two will fall apart.

Similarly, for the Olympic rings



there is a circle whose removing leads the remaining four to fall apart. Which one?

One way to study such questions mathematically is to assign some algebraic object, for instance, a polynomial (or a group, if you have already taken abstract algebra), to a picture of knotted rope in an invariant way. That is, if we move the rope around and get a different picture, then the objects corresponding to the two pictures must be the same (polynomials are equal, groups are isomorphic etc.).

During the program we are going to study such invariants and relate them to combinatorial graphs. Knots and graphs play a very important role in physics (quantum field theory) where they appear as Feynman diagrams, in biology where knots and graphs model protein structures, in computer science where they represent certain types of networks, and in many other areas.

The whole group will meet twice a week (Monday, Wednesday 2:30–3:30 PM) to discuss general theory and techniques. During the first weeks I am planning give a necessary theoretical background and state the problems in every small project. In each project will be some open problems which solutions is unknown so far. I expect a small group of about 3 students working on each project. I plan to meet with each small research group about one more time per week. On these small meeting we will discuss specific problems, methods, solutions related to the specific projects.