Tutte Polynomial of Signed Graphs and its Categorification

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Signed Graphs

Definition

A signed graph $\Sigma = (\Gamma, \sigma)$ is a graph $\Gamma = (V, E)$ and a signature $\sigma \colon E \to \{\pm 1\}$.

Balance

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A component W of a signed graph Σ is called balanced if the sign of each cycle of W is 1. A signed graph Σ is called balanced if each component of W is balanced.

Signed Colorings

Definition

A k-coloring of a signed graph Σ is a function $\gamma\colon V \to \{0, \pm 1, \pm 2, \ldots, \pm k\}$. We may exclude 0, and we will call such colorings zero-free colorings.

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Definition

A coloring of a signed graph Σ is called proper if for every edge $e = \{v, w\} \in E$, $\gamma(v)\sigma(e) \neq \gamma(w)$.

Coloring

Definition

A switching function for a signed graph Σ is a function $\zeta\colon V \to \{\pm 1\}$. The switched signature σ^ζ is defined by $\sigma^\zeta(e) = \zeta(v)\sigma(e)\zeta(w)$, where the edge e has endpoints v and w, and the switched signed graph is $\Sigma^\zeta = (\Gamma, \sigma^\zeta)$. The switched coloring γ^ζ is defined by $\gamma^\zeta(v) = \gamma(v)\zeta(v)$.

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Proposition

Switching does not alter the balance of cycles or the set of properly colored edges of a coloration.

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Say $\Sigma \sim \Sigma'$ if there is a switching function ζ such that $\Sigma^\zeta = \Sigma'$.

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Proposition

The relation $\sim$ defined above is an equivalence relation. We will refer to the equivalences classes of $\sim$ as the switching classes of a signed graph Σ.

Contractions

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Proposition

The resulting switching class of a contraction is unique.

Chromatic Polynomials

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Of course, from the definitions it is not obvious that these are, in fact, polynomials.

Deletion-Contraction Properties of χ_Σ and χ^*_Σ

Deletion-Contraction Properties

- $\chi_\Sigma = \chi_{\Sigma \setminus e} - \chi_{\Sigma / e}$
- $\chi^*_\Sigma = \chi^*_{\Sigma \setminus e} - \chi^*_{\Sigma / e}$ (e not a negative loop)
- $\chi^*_\Sigma = \chi^*_{\Sigma / e}$ (e a negative loop)

Matroid Structure

Given a subset of edges $F \subseteq E$ of a signed graph $\Sigma = (V, E, \sigma)$, we will identify F with the spanning subgraph $(V, F, \sigma|_F)$.

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Definition

- $k(F) = \#$ components of F
- $b(F) = \#$ balanced components of F
- $u(F) = \#$ unbalanced components of $F = k(F) - b(F)$
- $r(F) = |V| - b(F)$
- $n(F) = |F| - r(F)$

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The function $r(F)$ is a rank function that gives a matroid structure to the edge set of a signed graph (the function $n(F)$ is the nullity).

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$$\chi^*_\Sigma(\lambda) = \sum_{\substack{F \subseteq E \\ F \text{ balanced}}} (-1)^{|F|} \lambda^{b(F)}$$

Tutte Polynomial

Definition

Tutte Polynomial (of a signed graph Σ):

$$T_\Sigma(x,y) = \sum_{F \subseteq E} (x-1)^{r(E)-r(F)} (y-1)^{n(F)}$$

Signed Tutte Polynomial

We introduce the following generalization.

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$$T_\Sigma(x,y,z) = \sum_{F \subseteq E} (x-1)^{r(E)-r(F)}(y-1)^{n(F)}(z-1)^{u(F)}$$

An equivalent form is given by the following polynomial.

Definition

$$\widetilde{T}_\Sigma(x,y,z) = \sum_{F \subseteq E} (-1)^{|F|}(1+x)^{b(F)}(1+y)^{n(F)}(1+z)^{u(F)}$$

Deletion-Contraction Properties of $T_\Sigma(x, y, z)$

Deletion-Contraction Properties

- $T_\Sigma = T_{\Sigma \setminus e} + T_{\Sigma / e}$ (e not a loop or a bridge)
- $T_\Sigma = x T_{\Sigma \setminus e}$ (e a bridge)
- $T_\Sigma = y T_{\Sigma / e}$ (e a positive loop)

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- $\widetilde{T}_\Sigma(\lambda - 1, 0, 0) = \chi_\Sigma(\lambda)$
- $\widetilde{T}_\Sigma(\lambda - 1, 0, -1) = \chi^*_\Sigma(\lambda)$

Categorifying the Signed Tutte Polynomial

- Our original polynomial:

$$T_{\Sigma}(x, y, z) = \sum_{F \subseteq E} (x-1)^{r(E)-r(F)} (y-1)^{n(F)} (z-1)^{u(F)}$$

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- By change of variables and rearranging:

$$\widetilde{T}_{\Sigma}(x,y,z) = \sum_{F \subseteq E} (-1)^{|F|}(x+1)^{b(F)}(y+1)^{n(F)}(z+1)^{u(F)}$$

Chain groups

- We use truncated polynomial algebras $\mathcal{A} = \mathbb{Q}[a]/(a^2)$, $\mathcal{B} = \mathbb{Q}[b]/(b^2)$, $\mathcal{C} = \mathbb{Q}[c]/(c^2)$
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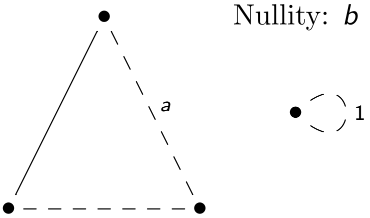

This gives the vector $a \otimes b \otimes 1_{\mathcal{C}} \in C_F = \mathcal{A} \otimes \mathcal{B} \otimes \mathcal{C}$

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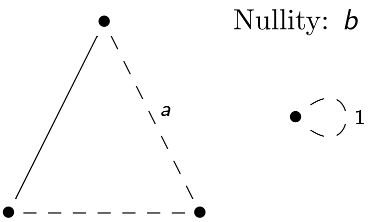
- Our chain groups are

$$C^i = \bigoplus_{\substack{F \subseteq E \\ |F| = i}} C_F$$



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Graded spaces and q-dim

The degree of a simple tensor $\mathbf{a} = a_1 \otimes \cdots \otimes a_n \in \mathcal{A}^{\otimes n}$, $a_i \in \{1, a\}$, is the number of occurrences of a in the n-tuple $(a_1, \ldots, a_n)$, and likewise for simple tensors in $\mathcal{B}^{\otimes n}$ and $\mathcal{C}^{\otimes n}$.

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Definition

For a triply graded vector space $V = \bigoplus_{i,j,k \in \mathbb{Z}} V_{i,j,k}$, where $V_{i,j,k}$ is the subspace generated by all vectors of $\mathcal{A}$-degree i, $\mathcal{B}$-degree j, and $\mathcal{C}$-degree k,
$q\text{-dim } V = \sum_{i,j,k \in \mathbb{Z}} x^i y^j z^k \dim V_{i,j,k}$

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q-dim $V = \sum_{i,j,k \in \mathbb{Z}} x^i y^j z^k \dim V_{i,j,k}$

Example: $q\text{-dim}\,\mathcal{A} = 1 + x$, $q\text{-dim}\,\mathcal{B} \oplus \mathcal{C} = 2 + y + z$, and $q\text{-dim}\,\mathcal{A} \otimes \mathcal{C} = (1 + x)(1 + z)$.

Boundary maps

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- We define the boundary maps from C_F to $C_{F \cup \{e\}}$ $(e \notin F)$ based on the labelled graph components. These induce the boundary maps from C_i to C_{i+1}.
- We look at what happens when we add the edge e to the subgraph F.

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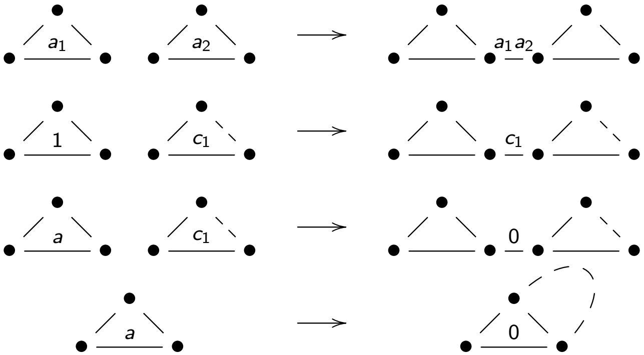
- If adding e causes a component K to become unbalanced, a label of $1 \in \mathcal{A}$ maps to $1 \in \mathcal{C}$, and a maps to 0.

Boundary maps: Balanced components



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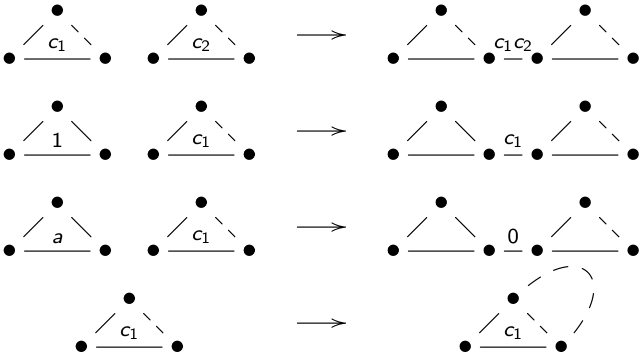
- If an unbalanced component is joined with a balanced component with label 1, the label is preserved; if the balanced component has label a, the label is sent to 0.
- If an unbalanced component is unchanged or only an edge is added (no vertices), its label is unchanged.

Boundary maps: Unbalanced components



Boundary maps: Final details

- Whenever the nullity of $F \cup \{e\}$ is greater than $n(F)$, the boundary map on the $\mathcal{B}$ part of C_F sends $\mathbf{b} \in \mathcal{B}^{\otimes n(F)}$ to $\mathbf{b} \otimes 1 \in \mathcal{B}^{\otimes n(F \cup \{e\})}$.

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- Finally, some of the maps $C_F \to C_{F \cup \{e\}}$ must be negated to ensure that $d_{n+1} \circ d_n = 0$ for all boundary maps d_n.

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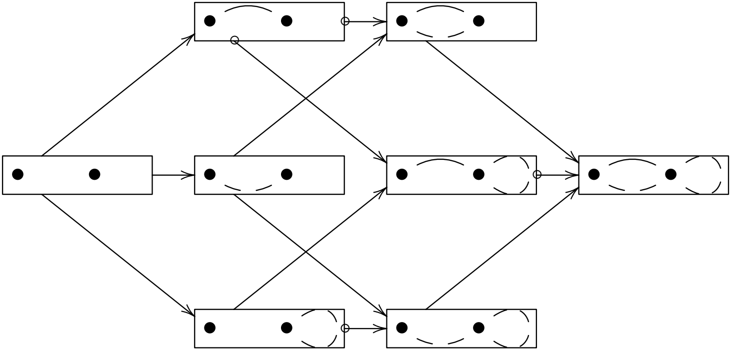
- Finally, some of the maps $C_F \to C_{F \cup \{e\}}$ must be negated to ensure that $d_{n+1} \circ d_n = 0$ for all boundary maps d_n.
- The Euler characteristic of our homology is

$$\sum_{i=0}^{\infty} (-1)^i \, q\text{-dim}\, H^i = \sum_{i=0}^{\infty} (-1)^i \, q\text{-dim}\, C^i$$

$$= \sum_{F \subseteq E} (-1)^{|F|} (x+1)^{b(F)} (y+1)^{n(F)} (z+1)^{u(F)} = \widetilde{T}_{\Sigma}(x, y, z)$$

Example of the Signed Tutte Homology



$\mathcal{A} \otimes \mathcal{A}$ $\quad$ $\mathcal{A} \oplus \mathcal{A} \oplus (\mathcal{A} \otimes \mathcal{C})$ $\quad$ $\mathcal{C} \oplus \mathcal{C} \oplus \mathcal{C}$ $\quad$ $\mathcal{B} \otimes \mathcal{C}$

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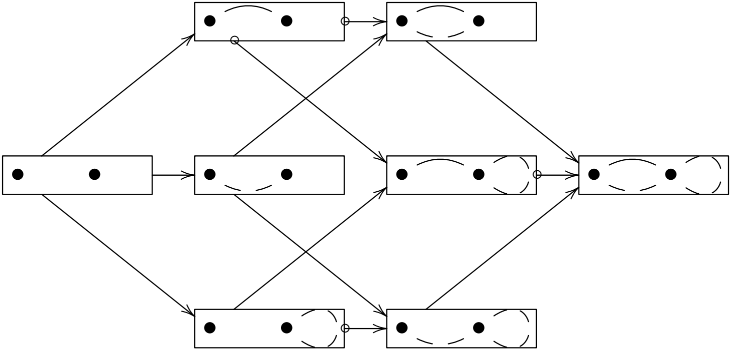

$a \otimes 1_{\mathcal{A}}$ $\quad (a, a, a \otimes 1_{\mathcal{C}})$ $\quad 0$

Signed Chromatic Homologies

Recall

$$\chi_\Sigma(2k+1) = \sum_{F \subseteq E} (-1)^{|F|} \lambda^{b(F)}$$

$$\chi^*_\Sigma(2k) = \sum_{\substack{F \subseteq E \\ \text{F balanced}}} (-1)^{|F|} \lambda^{b(F)}$$

Corresponding to these two chromatic polynomials, there are two signed chromatic homologies, defined similarly to our signed Tutte homology.

Full Chromatic Homology

- The full chromatic homology is a “sub-homology” - each chain group is (isomorphic to) a subspace of a corresponding chain group from our signed Tutte homology.

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- The full chromatic homology has chain groups built from $Ch_F = \mathcal{A}^{\otimes b(F)}$. Compare to our Tutte chain groups $C_F = \mathcal{A}^{\otimes b(F)} \otimes \mathcal{B}^{\otimes n(F)} \otimes \mathcal{C}^{\otimes u(F)}$.

Full Chromatic Homology

- The full chromatic homology is a "sub-homology" - each chain group is (isomorphic to) a subspace of a corresponding chain group from our signed Tutte homology.
- The full chromatic homology has chain groups built from $Ch_F = \mathcal{A}^{\otimes b(F)}$. Compare to our Tutte chain groups $C_F = \mathcal{A}^{\otimes b(F)} \otimes \mathcal{B}^{\otimes n(F)} \otimes \mathcal{C}^{\otimes u(F)}$.
- Moreover, the restriction of the Tutte boundary maps gives the chromatic boundary maps

Zero-Free Chromatic Homology

- The zero-free chromatic homology is built from spaces Ch_F^*, which are $\mathcal{A}^{\otimes b(F)}$ when F is balanced, and the zero vector space when F is unbalanced.

Zero-Free Chromatic Homology

- The zero-free chromatic homology is built from spaces Ch_F^*, which are $\mathcal{A}^{\otimes b(F)}$ when F is balanced, and the zero vector space when F is unbalanced.
- Similar to the full chromatic homology, this appears as a quotient of the signed Tutte homology.

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- In particular, we hoped to expand Thistlethwaite result connecting the Jones polynomial of alternating knots and the Tutte polynomial for ordinary graphs to general knots and signed graphs.
- This polynomial failed in that regard, and there was no immediate connection to the Khovanov homology