Knots and Graphs

Working Group [Summer 2013]

MATH 4193, class number 13323

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RESEARCH PROJECTS

Project 1. ***Parity for virtual links.*** (Duncan Clark, Adam Funke, Aaron Wong)

The Jones polynomial for virtual links can be twisted using so called *parity*, see [KK, Man]. The goal of this project is to try to define the similar notion for ribbon graph and generalize the Thistlethwaite theorem from [BBC] to ribbon graphs with parity.

Project 2. ***Weighted chromatic polynomial.*** (Peter Tian, Ji Hoon Chun, Isaac Smith)

In [St1] R. Stanley introduced a generalization of the chromatic polynomial of a graph in terms of multivariable symmetric functions and state a few conjectures about it. It was observed in [NW] that this generalization is equivalent to so-called *weighted chromatic polynomial* which previously occur in knot theory. It deals with weighted graphs whose vertices have integer weights which are additive under contraction of an edge. One of Stanley's conjecture claims that this polynomial distinguish trees. We are planing to work on it trying to transform it to weighted chromatic polynomial. Another possible direction of this project is the Stanley-Gasharov conjecture from [Ga, St2].

Project 3. ***Chromatic polynomial of signed graphs.*** (Ben O'Connor, Andrew Krieger)

Signed graphs are usual graphs with signs $\pm$ assigned to its edges. These graphs are naturally appear in relation to knot theory. There are two generalizations of the chromatic polynomial to signed graphs [BZ, Z1, Z2]. The goal of the project is to categorify these polynomials in a sense of the Khovanov type homology theory. For unsigned graphs the chromatic polynomial was categorified in [HGR]. However the results of [CCR] shows that the chromatic polynomial of an (unsigned) graph completely determines its homology groups. The hope is that under the "right" categorification for signed graphs these groups will contain more information than the polynomials. Also these groups might be tighter related to the Khovanov homology of knots.

Project 4. ***Simplicial flow polynomial.*** (Bradley Burdick, Noah Taylor)

The higher dimensional Tutte polynomial is an invariant for cell complexes introduced in [KR]. It was studied in a previous year group by Carlos Bajo, Bradley Burdick [BBC]. In the classical situation for graphs the Tutte polynomial specializes to the so called *flow polynomial* (see, for example [Bo]). Flow polynomial for simplicial complexes was introduced recently in [BK]. One goal of this project is to relate their flow polynomial to the Tutte-Krushkal-Renardy polynomial. Another goal of this project is to understand the relation between various higher dimension notions of spanning trees and the Tutte polynomial from [CCK, K, P]. Also with relation to the project #3 it would be interesting to determine a flow polynomial of signed graphs.

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