Knots and Graphs

Working Group [Summer 2014]

MATH 4193, class number 16791

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RESEARCH PROJECTS

Project 1. ***Global differential geometry.*** (Tyler Friesen, Boming Jia)

This project project is a development of some topics of my differential geometry 5540H course.

The classical theorem of H. Hopf and Fary-Milnor [doCarmo] relate the integral of the curvature of a curve with a topological invariant of the curve. Namely the Hopf theorem [doCarmo, Sec.5.7, Theorem 2, p.396] claim that the integral of the curvature of a plane immersed curve is equal to the rotation index of the curve (see also my handout to Math 5540H: `https://people.math.osu.edu/chmutov.1/13-14/m5540H/glob-pl-curves.pdf`.) The Fary-Milnor theorem [doCarmo, Sec.5.7, Theorem 4, p.402] states that the integral of a curve non-trivially knotted in $\mathbb{R}^3$ is greater or equal to 4π. Recently it was found the other theorems of this type [LP] for planar curves. The aim of the project is to try to found an analogous theorem for spacial curves.

Project 2. ***Weighted chromatic polynomial.*** (Isaac Smith, Zane Smith, Anthony Ciavarella, and Peter Tian)

In [St1] R. Stanley introduced a generalization of the chromatic polynomial of a graph in terms of multivariable symmetric functions and state a few conjectures about it. It was observed in [NW] that this generalization is equivalent to so-called *weighted chromatic polynomial* which previously occur in knot theory. It deals with weighted graphs whose vertices have integer weights which are additive under contraction of an edge. One of Stanley's conjecture claims that this polynomial distinguish trees. We are planing to work on it trying to transform it to weighted chromatic polynomial. Another possible direction of this project is the Stanley-Gasharov conjecture from [Ga, St2].

Project 3. ***Braess's paradox.*** (Ji Hoon Chun, Henry Tran, Alexander While)

This is a project about Braess's paradox which states that adding extra capacity to a network when the moving entities selfishly choose their route, can in some cases reduce overall performance. This is because the Nash equilibrium of such a system is not necessarily optimal. The main source is `Braess's paradox: http://en.wikipedia.org/wiki/Braess%27s_paradox`

The paradox is stated as follows:

"For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times."

Project 4. *Dimer model for knot invariants.* (Duncan Clark, Caleb Lehman)

Recently the dimer model from statistical physics and combinatorics was applied to knot theory [CT, CDR]. The goal of the project is to try to apply this approach to other polynomial invariants of links, in particular virtual links.

References

[CT] M. Cohen, M. Teicher, *Kauffman's clock lattice as a graph of perfect matchings: a formula for its height.* Preprint `arXiv:1211.2558 [math.GT]`.

[CDR] M. Cohen, O. Dasbach, H. Russell, *A twisted dimer model for knots*, Fund. Math.(2012). Preprint `arXiv:1010.5228 [math.GT]`.

[doCarmo] M.do Carmo, *Differential Geometry of Curves and Surfaces.* Prentice-Hall, 1976.

[Ga] V. Gasharov, *On Stanley's chromatic symmetric function and clawfree graphs*, Discrete Mathematics **205** (1999) 229–234.

[LP] S. Lanzat, M. Polyak, *Integrating curvature: from Umlaufsatz to J^+ invariant.* Preprint `arXiv:1108.4288 [math.GT]`.

[NW] S. Noble, D. Welsh, *A weighted graph polynomial from chromatic invariants of knots*, Annales de l'institut Fourier **49**(3) (1999) 1057–1087.

[St1] R. Stanley, *A symmetric function generalization of the chromatic polynomial of a graph*, Adv. Math., **111**(1) (1995) 166-194.

[St2] R. Stanley, *A chromatic symmetric function conjecture*, slides of a talk the AMS Winter Meeting, Boston, 7 January 2012. `http://www-math.mit.edu/~rstan/transparencies/3plus1.pdf`.