Knots and Graphs
Working Group [Summer 2015]
MATH 4193, class number 17015
Instructor: **Sergei Chmutov**

RESEARCH PROJECTS

Project 1. ***Discrete Four Vertex Theorem.*** (Curtis Franks, Boming Jia, Johann Miller)

The classical four vertex theorem claims that a simple closed convex smooth curve, oval, has at least four critical point of the curvature function, vertices. There are several discrete versions of this theorem. Most general of them is in [Tab]. From the other side, there is a general approach to discrete version of differential geometry based on the concept of oriented matroids [MPh, And]. The aim of the project is to try to put the discrete four vertex theorem into this general framework of combinatorial differential geometry.

Project 2. ***Hunting of Snarks.*** (Carl Ahlborg, Nicholas Hemleben, Zane Smith, Woodward Denver)

Snark is a fictional animal created by Lewis Carroll in his poem *The Hunting of the Snark*. In mathematics a *snark* is a connected trivalent graph whose edges connot be colored in three colors in a way that the three edges meeting at a vertex colored differently [Ga, Ch.23]. The smallest snark is the Petersen graph. W. T. Tutte conjectured that every snark has the Petersen graph as a minor. We will analyze the existing constructions of snarks, try to find new constructions and approach to the Tutte conjecture.

Project 3. ***Knots & Matroids.*** (Caleb Dilsavor, Rao Sujit, Alexander While)

Delta-matroids were introduced by Bouchet [Bou] in 1987. Recently it was realized [CMNR] that they play an important part in topology of ribbon graphs. One goal of this project is to try to construct a delta-matroid directly from a link diagram and possibly relate it to some classical polynomial invariants of links. Another goal is to try to relate delta-matroids with *transition matroids* of L. Traldi [Tr] which are related to knots and links.

Project 4. ***Quasi-trees.*** (Clark Duncan, Caleb Lehman, Murphy Daniel)

A quasi-tree is a ribbon graph with a single boundary component. This project is about quasi-tree expansion of the Krushkal polynomial found by Clark Butler few years ago [But] (see also an expository paper [Ch]). The goal is to try to find a kind of quasi-tree expansion for the Jones polynomial of a link.

References

[And]   L. Anderson, *Topology of combinatorial differential manifolds*, Topology **38**(1) (1999) 197–221.

[Bou]   A. Bouchet, *Greedy algorithm and symmetric matroids*, Math. Program. **38** (1987) 147-159.

[But]   C. Butler, *A quasi-tree expansion of the Krushkal polynomial*, Preprint `arXiv: 1205.0298 [math.CO]`.

[Ch]   S.Chmutov, *Topological Extensions of the Tutte Polynomial*. A draft of a chapter for the *Handbook of the Tutte Polynomial*.

[CMNR]   C. Chun, I. Moffatt, S. Noble, R. Rueckriemen, *Matroids, delta-matroids and embedded Graphs*, Preprint `arXiv:1403.0920 [math.CO].`

[Ga]   M. Gardner, *The Last Recreations: Hydras, Eggs, and Other Mathematical Mystifications*, Springer, (2007).

[MPh]   R. MacPherson, *Combinatorial dierential manifolds*, in Topological Methods in Modern Mathematics: a Symposium in Honor of John Milnor's Sixtieth Birthday, Publish or Perish, Berkeley, CA, (1993) 203–221.

[Tab]   S. Tabachnikov, *A Four Vertex Theorem for Polygons*, American Mathematical Monthly **107**(9) (2000) 830–833.

[Tr]   L. Traldi, *The transition matroid of a 4-regular graph: an introduction*, Preprint `arXiv:1307.8097 [math.CO].` To appear in the European Journal of Combinatorics.