Knots and Graphs

Working Group [Summer 2016]

MATH 4193, class number 17165

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RESEARCH PROJECTS

Project 1. ***Knots and links in spatial graphs.*** (Michael Crawshaw, Caleb Dilsavor, Grace Tian)

This is the title of a famous paper [CG] by John Conway and Cameron Gordon showing that every embedding of the complete graph K_6 in $\mathbb{R}^3$ contains at least one pair of linked triangles and every embedding of K_7 contains a non-trivial knot. An excellent exposition of this results see in [Ad, Ch.8]. A paper [FMMNN] is recent survey of this area.

The aim of the project is to try to relate the spatial knotedness and linkage of graphs with link invariants. For example, the Conway-Gordon theorem can be restated that for any embedding of K_6 there two cycles with the linking number at least 1. An example of the simplest question might be as follows. Is it true that for any embedding of K_7 there are two cycles with the linking number at least 2?

Project 2. ***Δ-matroids.*** (Zane Smith, Woodward Denver)

Δ-matroids were introduced by A.Bouchet [Bou] in 1987. Recently it was realized [CMNR] that they play an important part in topology of ribbon graphs.

The goal of this project is to study the structure of Δ-matroids. There are many usual matroids related to a Δ-matroid D. For instance, there are maximal and minimal matroids $D_{\max}$ and $D_{\min}$ associated with it. It is known that they are related by a *matroid perspective* $D_{\max} \to D_{\min}$ in sense of M. Las Vergnas [LV1, LV2]. Conjecturally any Δ-matroid consists of a collection of matroid perspectives. We will try to find the conditions on this collection to for forming a Δ-matroid.

Project 3. ***Dimer model for virtual knots.*** (Dan Brogan, Nicholas Hemleben, Jiaqi Liu)

Recently the dimer model from statistical physics and combinatorics was applied to knot theory [CT, CDR]. The goal of the project is to try to find an analogous approach to virtual links.

Project 4. ***Higher dimensional flow polynomial.*** (Desmond Coles, Nik Henderson, Aidan Howells)

This is a development of a project from previous years. The *flow polynomial* of graphs is a specialization of the Tutte polynomial (see, for example [Bo]). For simplicial complexes the Flow polynomial was introduced in [BK]. We will try to improve the results of B.Burdick [Bur] about nowhere-zero flows in the direction of F.Jaeger [Jae].

References

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