Knots and Graphs

Working Group [Summer 2018]

MATH 4193, class number 17324

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RESEARCH PROJECTS

Project 1. ***Quandles.*** (Will Hoffer, Adu Vengal, Vilas Winstein)

This is a continuation of the last year project. It deals with *skein relation* type invariants of knots colored in a few colors. It was based on the paper [Yang]. Last year we found a new link invariant with values in the finite field $\mathbb{F}_8$associated with a bicoloring of a link diagram. For tricoloring of diagrams, we found an invariant with values in a finite ring of 27 elements.

This year we are going to work on generalization of our results to link diagrams colored by an arbitrary *quandle*, that is an object from abstract algebra[EN]. Possibly we will come to work with quandle cocycle invariants.

Project 2. ***Symmetric Tutte polynomial and knots.*** (Jake Huryn, Neng Wang)

Classical Thistlethwaite's theorem [Th] claims that the Jones polynomial of a link is equal to a specialization of the Tutte polynomial of a graph constructed from a link diagram. A good reference about the Jones polynomial is [Ad, Ch.6]. The first reading about Thistlethwaite's theorem is [Ka1] with the subsequent reference [Ka2].

The Tutte polynomial is a generalization of the chromatic polynomial of a graph. There is a multivariable generalization of the chromatic polynomial, *Stanley's chtomatic symmetric function* [St1]. Later R. Stanley gave [St2] a symmetric generalization of the Tutte polynomial, see also [NW].

The goal of the project is look whether any specialization of the symmetric Tutte polynomial can provide a link invariant. This project may be related to the next one.

Project 3. ***Symmetric chromatic polynomial of signed graphs.*** (James Enouen, Eric Fawcett, Rushil Raghavan, Ishaan Shah)

Signed graphs are usual graphs with signs $\pm$ assigned to its edges. These graphs are naturally appear in relation to knot theory. There are two generalizations of the chromatic polynomial to signed graphs [Z1, Z2]. The goal of the project is to try to find a symmetric generalization of these polynomials in sense of R. Stanley [St1] from previous project. The hope is that it might be B_n-type symmetric function invariant under signed permutations.

Project 4. ***Reidemeister moves of virtual knots.*** (John Beck, Jack DePascale, Megan Fava, Juhee Park)

Recently M.-J. Jeong [Jeo] found a polynomial constructed from a Gauss diagram of a virtual knot which changes under the Reidemeiset moves in a very special way. This allows to use use this polynomial

for estimating the number of Reidemeister moves from one diagram of a knot to another one. We are going to study this paper, try to improve the estimates and try to generalize it for links.

References

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