Knots and Graphs

Working Group [Summer 2019]

MATH 4193, class number 15996

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RESEARCH PROJECTS

Project 1. ***Quandles.*** (James Enouen, Adu Vengal, Vilas Winstein)

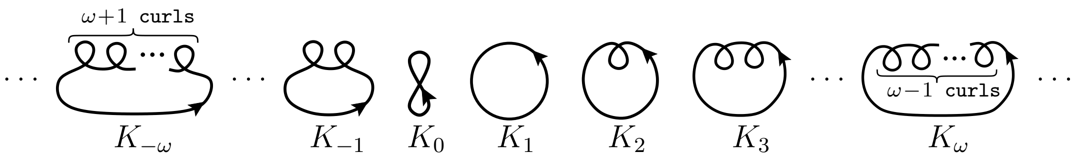
This is a continuation of the last year project. *(Bi)quandle* is an object from abstract algebra which is a set with some operations on its elements, see[EN]. The arcs of a link diagrams can be colored by elements of a quandle satisfying some natural relation at every crossing which involves the quandle operations. This leads to a notion of *(bi)quandle bracket*, invariant under the Reidemester moves. The goal of this year project is to try to categorify the biquandle bracket in a similar way the Khovanov homology categorifies the Jones polynomial.

Project 2. ***Symmetric functions.*** (Vlad Akavets, Noah Donald, Eric Fawcett, Hannah Hasan, Jake Huryn, Ishaan Shah)

This is also a continuation of the last year project dealing with *Stanley's chromatic symmetric function* [St1]. So the students know better what they are going to work on. I can suggest couple problems I would be interested in. First one is to find a symmetric generalization of the Tutte polynomial which would respect the duality of planar graphs. The second one is about symmetric generalization of Stanley's acyclic orientations theorem [St1, Theorem 3.3] to signed graph and B-symmetric chromatic polynomial the students worked with last year.

Project 3. ***Immersed plane curves.*** (Ankan Bhattacharya, Kat Husar, Dennis Sweeney, John White)

An *immersion* is a map $\gamma : S^1 \to \mathbb{R}^2$ of a circle to the plane such that its derivative is a non-zero vector for any point on S^1. The classical Whitney–Graustein theorem states that any immersed plane curve can be homotoped, that is smoothly deformed, to a canonical curve K_ω with the same winding number.

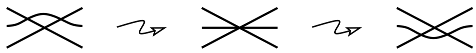


`Canonical curves` K_ω `with winding number` ω

A generic immersed curve has only double points as singularities. A knot diagram with the information of over-crossing and under-crossing forgotten gives an example of such a curve. A generic homotopy involves finitely many moves through non-generic curves having a self-tangency point or a triple point [Ar].

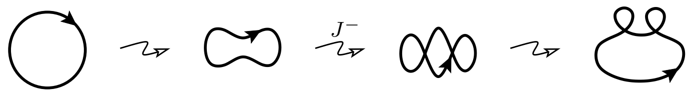
direct self-tangency J^+-move

inverse self-tangency J^--move



triple point St-move

For example, a circle oriented clockwise has winding number -1. So it should be a homotopy connecting it with K_{-1}. Here is one:

$\overline{K}_1 =$  $= K_{-1}$

homotopy of the clockwise oriented circle to K_{-1}

The *complexity* $c(K)$ of a curve K is the minimal number of these moves over all possible generic homotopies from the curve to a canonical curve.

T.Howik [No] found some quadratic estimates for the complexity of a curve with n double points. On the other hand H.-C. Chang and J. Erickson [CE] found the estimate $n^{3/2}$ for the complexity using slightly different set of moves. The goal of this project is to compare these two approaches to the complexity and try to improve the estimates. Perhaps the Arnold's invariants from [Ar] could be useful for this.

References

[Ar] V. I. Arnold, *Topological invariants of plane curves and caustics*, University Lecture Notes **5** (1994) AMS, Providence.

[CE] H.-C. Chang, J. Erickson, *Untangling planar curves*, Discrete & Computational Geometry **58**(4) (2017) 889–920.

[EN] M. Elhamdadi, S. Nelson, *Quandles. An introduction to the algebra of knots*, AMS, 2015.

[No] T. Nowik, *Complexity of plane and spherical curves*, Duke Mathematical Journal **148**(1) (2009) 107–118.

[St1] R. Stanley, *A symmetric function generalization of the chromatic polynomial of a graph*, Advances in Math. **111**(1) (1995) 166–194.