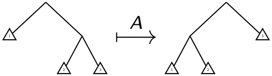
Computing in Thompson's Group

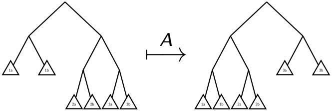
OSU Knots and Graphs

Summer 2020

Computing in Thompson's Group

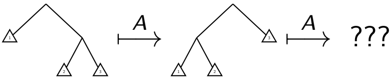
- What does a pair of trees represent?
  - 

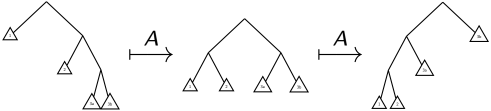
  - Leave those three subtrees the same, but 'rotate' the way they connect together.
  - An example with nontrivial subtrees:

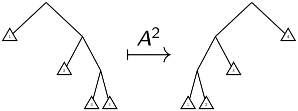


- It may help to think of the subtrees as infinte; this is an "automorphism" of the complete infinite binary tree.

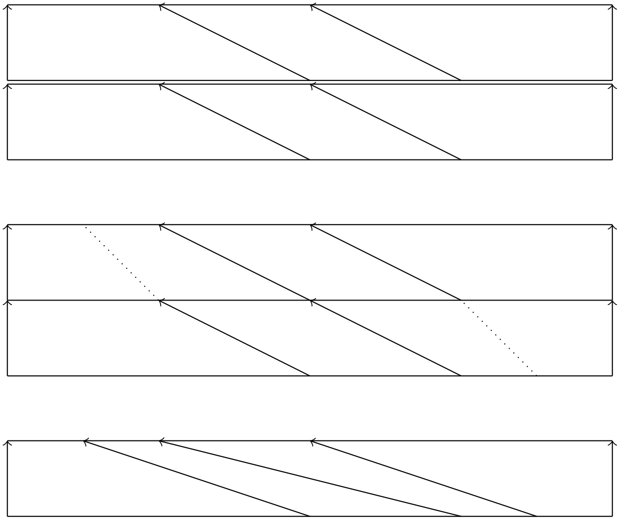
A Complication

- Concern: What if those subtrees don't exist?
- For example, try to compute A^2:
- 

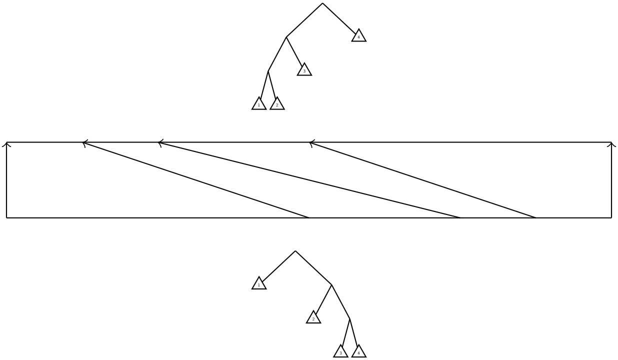
- Resolution: Subdivide more until they do.
- 

- Conclusion: 

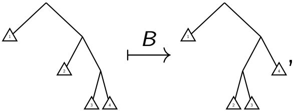
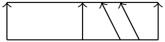
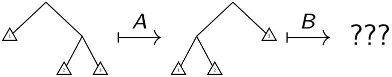
Double-Check

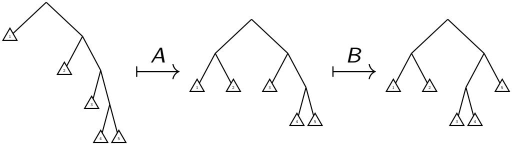


Double-Check



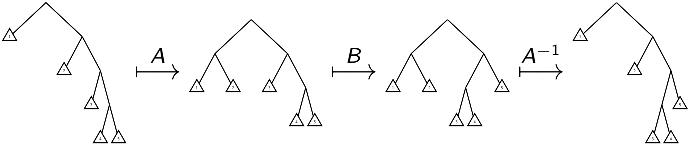
Another example

- Define B with  , or 
- Let's compute BA (A then B).
- 

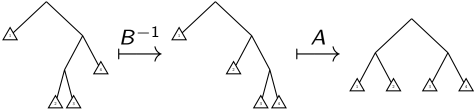
- 

Verifying a Defining Relation

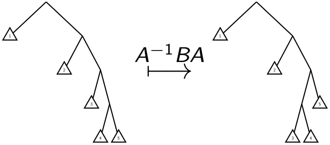
- $F = \langle A, B \,:\, [AB^{-1}, A^{-1}BA] = [AB^{-1}, A^{-2}BA^2] = 1 \rangle$
- Let's verify that AB^{-1} and $A^{-1}BA$ commute.
- $A^{-1}BA$:

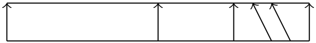
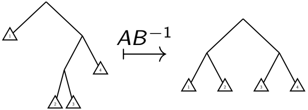


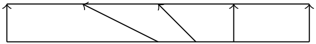
- AB^{-1}:



Verifying a Defining Relation

- 


- 


- Disjoint actions $\implies$ commute
- Other relation is similar, just one layer deeper.
- This shows that these relations hold, but surprisingly, these are *all* of the relations needed.

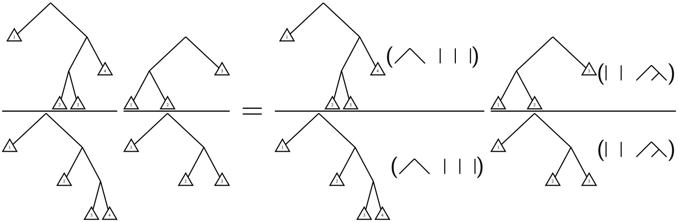
An Analogy: The group $\mathbb{Q}^{\times}_{>0}$

- $10 \overset{3/10}{\longmapsto} 3$.
  - It follows that $70 \overset{3/10}{\longmapsto} 21$.
- To multiply $\frac{3}{10} \cdot \frac{14}{11}$, we have

$$11 \overset{14/11}{\longmapsto} 14 \overset{3/10}{\longmapsto} \;???$$

- But $55 \overset{14/11}{\longmapsto} 70 \overset{3/10}{\longmapsto} 21$, so $\frac{3}{10} \cdot \frac{14}{11} = \frac{21}{55}$.
- In summary, $\frac{3}{10} \cdot \frac{14}{11} = \frac{3 \cdot 7}{10 \cdot 7} \cdot \frac{14 \cdot 5}{11 \cdot 5} = \frac{21}{70} \cdot \frac{70}{55} = \frac{21}{55}$.
- Do the same things with trees!?

Computing with Tree-Fractions

- Compare to $\frac{3}{10} \cdot \frac{14}{11} = \frac{3\cdot7}{10\cdot7} \cdot \frac{14\cdot5}{11\cdot5} = \frac{21}{70} \cdot \frac{70}{55} = \frac{21}{55}$.
- $BA =$ 
- $=$