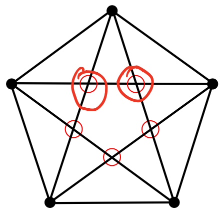
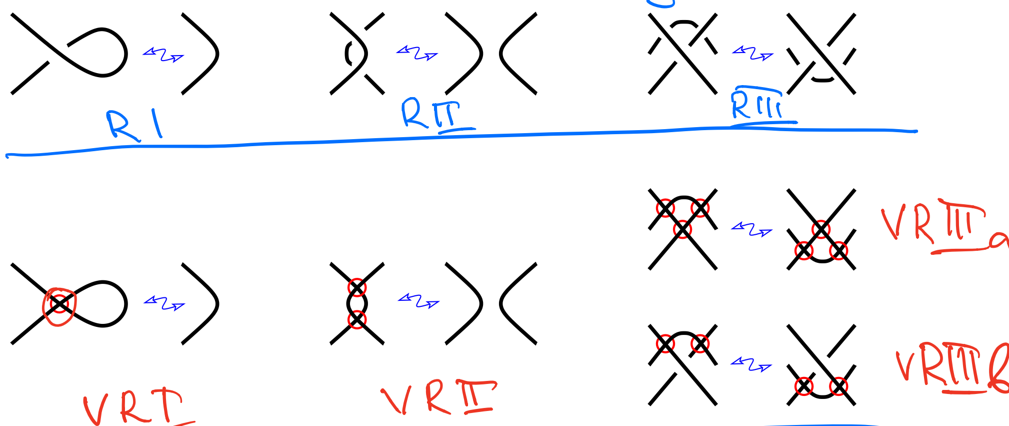
(Virtual) links [Ka2].

Virtual crossings

$K_5 =$

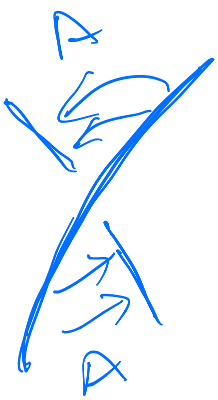


Reidemeister moves



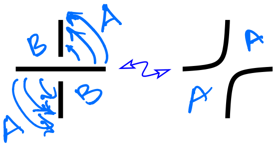
The Kauffman bracket and the Jones polynomial [Ka1]

.

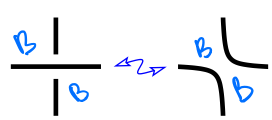


Let L be a link diagram.

A-splitting:



B-splitting:



A *state* S is a choice of either A- or B-splitting at every classical crossing.

$\alpha(S) = \#(\text{of } A\text{-splittings in } S)$

$\beta(S) = \#(\text{of } B\text{-splittings in } S)$

$\delta(S) = \#(\text{of circles in } S)$

Kauffman bracket

$$\boxed{[L](A,B,d) \;:=\; \sum_S A^{\alpha(S)}\, B^{\beta(S)}\, d^{\delta(S)-1}}$$

B d

$$\boxed{J_L(t) \;:=\; (-1)^{w(L)} t^{3w(L)/4} [L](t^{-1/4}, t^{1/4}, -t^{1/2} - t^{-1/2})}$$

A

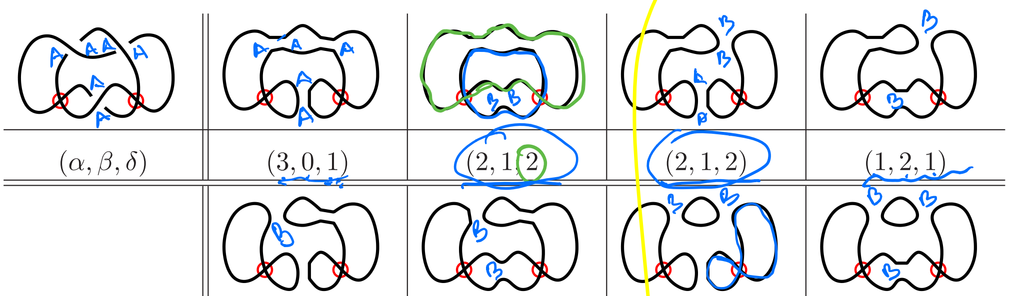
Jones pol.

$A = t^{-1/4}$

$B = t^{1/4}$

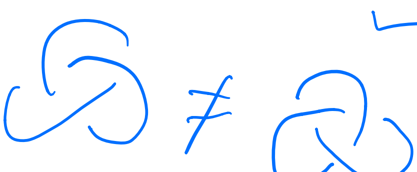
$d = -t^{1/2} - t^{-1/2}$

Example

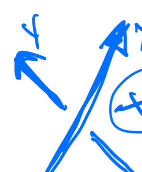


(α,β,δ)	$(3,0,1)$	$(2,1,2)$	$(2,1,2)$	$(1,2,1)$
	$(2,1,2)$	$(1,2,1)$	$(1,2,3)$	$(0,3,2)$

$[L] = A^3 + 3A^2Bd + 2AB^2 + AB^2d^2 + B^3d$; $\quad \boxed{J_L(t) = 1}$



writhe of L

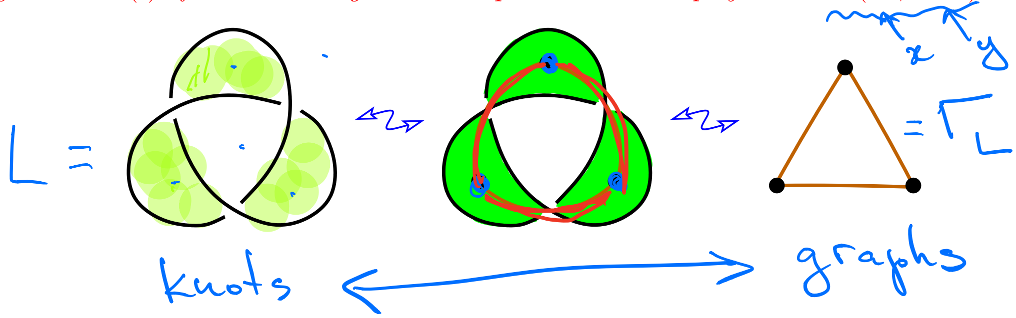


$(+1) = \varepsilon$

$(-1) = \varepsilon$

$w(L) = \sum_c \varepsilon(c)$

Thistlethwaite's Theorem [Ka1] *Up to a sign and multiplication by a power of t the Jones polynomial $J_L(t)$ of an alternating link L is equal to the Tutte polynomial $T_\Gamma(-t, -t^{-1})$.*



References

[Ka1] L. H. Kauffman, *New invariants in knot theory*, Amer. Math. Monthly **95** (1988) 195–242.
[Ka2] L. Kauffman, *Virtual knot theory*, European Journal of Combinatorics, **20** (1999) 663–690.