The Las Vergnas polynomial
Reference: M. Las Vergnas [LV].

**Matroid perspectives.**
A bijection \( M \to M' \) is called *matroid perspective* if any circuit of \( M \) is mapped to a union of circuits of \( M' \). Equivalently, \( r_M(X) - r_M(Y) \geq r_{M'}(X) - r_{M'}(Y) \) for all \( Y \subseteq X \).

**Example.**
For graphs \( G \) and \( G^* \) dually embedded in a surface, then the map of the bond matroid of \( G^* \) onto the circuit matroid of \( G \), \( B(G^*) \to C(G) \), is a matroid perspective.

**Definition.**
\[
T_{M \to M'}(x, y, z) := \sum_{X \subseteq M} (x - 1)^{r(M') - r_M(X)} (y - 1)^{n(M) - r_M(X)} z^{-(r(M') - r_M(X))}
\]

**Properties.**
\[
T_M(x, y) = T_{M \to M'}(x, y, z); \quad T_M(x, y) = T_{M \to M'}(x, y, x - 1); \quad T_{M'}(x, y) = (y - 1)^{r(M') - r(M')} T_{M \to M'}(x, y, \frac{1}{y - 1});
\]

**Ribbon graphs**

**Definition.** A *ribbon graph* \( G \) is a surface (possibly non-orientable) with boundary, represented as the union of two sets of closed topological discs called *vertices* \( V(G) \) and *edges* \( E(G) \), satisfying the following conditions:

- these vertices and edges intersect by disjoint line segments;
- each such line segment lies on the boundary of precisely one vertex and precisely one edge;
- every edge contains exactly two such line segments.

\[
R_G(\{x_e, y_e\}, X, Y, Z) := \sum_{F \subseteq G} \left( \prod_{e \in F} x_e \right) \left( \prod_{e \in F} y_e \right) X^{r(G) - r(F)} Y^{n(F)} Z^{k(F) - bc(F) + n(F)}
\]

For signed graphs, we set \( x_+ = 1, \quad x_- = (X/Y)^{1/2}, \quad y_+ = 1, \quad y_- = (Y/X)^{1/2} \).
Example.

\[
R_G(X, Y, Z) = X + 2 + Y + XYZ^2 + 2YZ + Y^2Z
\]

Properties.

\[
R_G = x_eR_{G/e} + y_eR_{G-e} \quad \text{if } e \text{ is ordinary, that is neither a bridge nor a loop},
\]

\[
R_G = (x_e + Xy_e)R_{G/e} \quad \text{if } e \text{ is a bridge}.
\]

\[
R_{G_1 \sqcup G_2} = R_{G_1} \cdot R_{G_2}
\]

References
