Project 1. **Virtual links and arrow polynomial.** (Ben O’Connor, Erin Zwick, Noah Taylor)

Virtual link diagrams, besides the classical crossings with the information of which strand goes on the top and which one goes on bottom provided, may have also virtual crossings where this information is not specified. In such form, virtual links were introduced by L. Kauffman in [Ka]. Independently, at about the same time, virtual links were introduced by M. Goussarov, M. Polyak, O. Viro [GPV] in terms of Gauss diagrams. For virtual links there is a generalization of the Jones polynomial, the arrow polynomial [DK], which is defined within the Kauffman approach. L. Kauffman announced [Ka1] a more general invariant of virtual knots taking values in the set of graphs. Last year Ben O’Connor found a gap in this work. The goal of the project is to try to fill in the gap and/or to try to generalize the arrow polynomial.

Project 2. **Higher dimensional Tutte polynomial.** (Jon Michel, Joel Arter, Kailin Huang, Nick Kosar)

The higher dimensional Tutte polynomial is an invariant for cell complexes introduced in [KR]. It was studied last year by Carlos Bajo, Bradley Burdick [BBC]. In the classical situation for graphs the Tutte polynomial specializes to the so called flow polynomial (see, for example [Bo]). Flow polynomial for simplicial complexes was introduced recently in [BK]. One goal of this project is to relate their flow polynomial to the Tutte-Krushkal-Renardy polynomial.

The another goal of this project is to understand the relation of the modified Tutte-Krushkal-Renardy polynomial from [BBC] with the Tutte polynomial of arithmetic matroids from [DAM1] and their arithmetic flow polynomial from [DAM2].

Project 3. **Matrix-quasi-tree theorem for ribbon graphs.** (Andrew Krieger, Isaac Smith, Amy Weisman)

This is a continuation of the project from the last summer.

A classical matrix-tree theorem expresses the determinant of some matrix constructed from a graph (principal minor of the Laplacian) as a sum over all spanning trees of the graph. For ribbon graphs instead of spanning trees it is more natural to consider spanning quasi-trees [CKS]. The project is intended to search for an appropriate matrix-tree type theorem for quasi-trees. That is to look for a matrix whose determinant would give the generating function of quasi-trees in a given ribbon graph. Some nice results in this direction were obtained by Patrick Schnell last summer. They need to be cleaned up and related to article [DFKLS].
Project 4. Planar graphs. (Ji Hoon Chun, Robin Baidya, Tyler Friesen, Peter Tian)

This is also a continuation of the project from the last summer.

The classical Kuratowski theorem states that a graph is planar if and only if it has no subgraph homeomorphic to $K_5$ or $K_{3,3}$ (see, for example, [Har]). Recently, some interest for planarity of graphs with crossing structure, $X$-graphs, appeared in knot theory [Va]. An $X$-graph is a regular graph each vertex of which has degree 4 and the four edges meeting at a vertex are parted into two pairs of two edges each. An $X$-embedding of an $X$-graph is an embedding of the graph to a surface when the partition form a crossing. V. Vassiliev [Va] formulated a conjecture stating that a plane $X$-embedding of an $X$-graph exists if and only if it does not contain two circuits intersection at a single vertex. The conjecture was proved in [Man1]. In this project we will try to simplify the proof of this conjecture, relate it to the other planarity criteria, and study a general formula for the minimal $X$-genus of $X$-graphs from [Man2].

References


