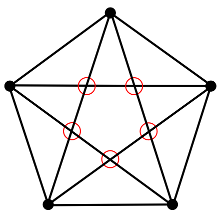
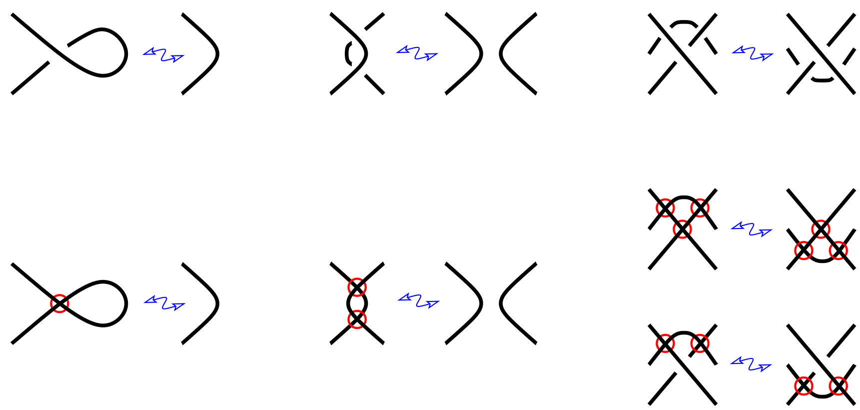
(Virtual) links [Ka2].

Virtual crossings

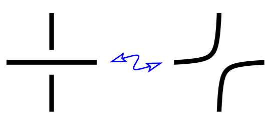


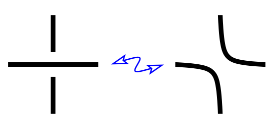
Reidemeister moves



The Kauffman bracket and the Jones polynomial [Ka1]

Let L be a link diagram.

A-splitting: 

B-splitting: 

A *state* S is a choice of either A- or B-splitting at every classical crossing.

$\alpha(S) = \#(\text{of } A\text{-splittings in } S)$

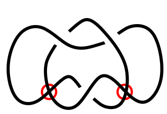
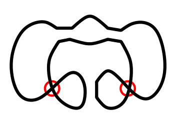
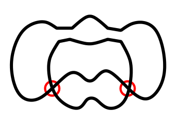
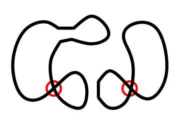
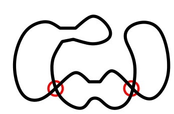
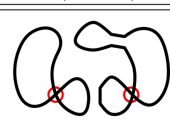
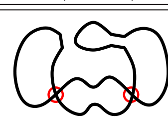
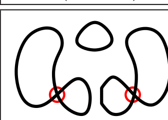
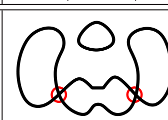
$\beta(S) = \#(\text{of } B\text{-splittings in } S)$

$\delta(S) = \#(\text{of circles in } S)$

$$\boxed{[L](A, B, d) \ := \sum_{S} A^{\alpha(S)}\, B^{\beta(S)}\, d^{\delta(S)-1}}$$

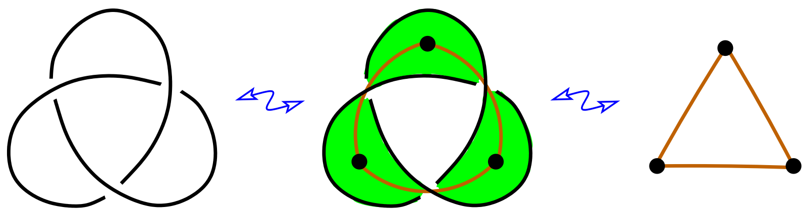
$$\boxed{J_L(t) \ := (-1)^{w(L)} t^{3w(L)/4} [L](t^{-1/4}, t^{1/4}, -t^{1/2} - t^{-1/2})}$$

Example

				
(α, β, δ)	$(3, 0, 1)$	$(2, 1, 2)$	$(2, 1, 2)$	$(1, 2, 1)$
				
	$(2, 1, 2)$	$(1, 2, 1)$	$(1, 2, 3)$	$(0, 3, 2)$

$$[L] = A^3 + 3A^2Bd + 2AB^2 + AB^2d^2 + B^3d; \qquad J_L(t) = 1$$

Thistlethwaite's Theorem [Ka1] *Up to a sign and multiplication by a power of t the Jones polynomial $J_L(t)$ of an alternating link L is equal to the Tutte polynomial $T_\Gamma(-t, -t^{-1})$.*



References

[Ka1] L. H. Kauffman, *New invariants in knot theory*, Amer. Math. Monthly **95** (1988) 195–242.

[Ka2] L. Kauffman, *Virtual knot theory*, European Journal of Combinatorics, **20** (1999) 663–690.