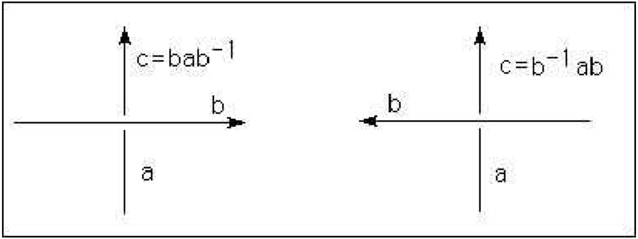
The groups of knots

Kabir Belgikar, Calvin Forsee, Wo Wu, OSU.

Mentor: Sergei Chmutov

July 11th, 2022.

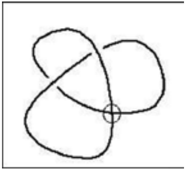
The group of a classical knot can be described by generators and relations, with one generator for each arc in the diagram and one relation for each crossing. The relation at a crossing depends upon the type of the crossing and is either of the form $c = b^{-1}ab$ or $c = bab^{-1}$.



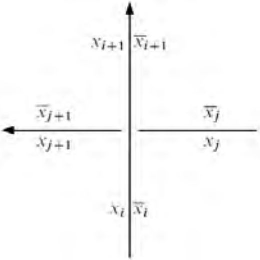
An arc is a segment from an undercrossing to the next undercrossing. You can think a knot group in the form of $\langle t_1, ..., t_p | r_1, ..., r_q \rangle$, where t_i are letters assigned to each arc and r_i are relations.

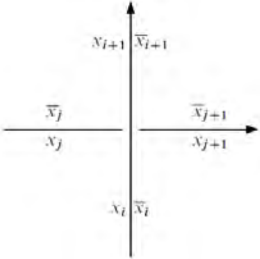
The group of a virtual knot is basically the same as a classical knot, all you need to do is to ignore the virtual crossing.

There are virtual knots that are not equivalent to the unknot but have the same group as the unknot, one example is the following one.



For twisted knots, it's a bit more complicated. It has a letter for each side of each end of an arc and has more relations.



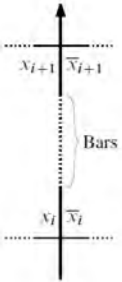


The relations at crossings are:

$$x_{i+1} = x_i, \qquad x_{j+1} = x_i^{-1} x_j x_i,$$

$$\overline{x}_{i+1} = \overline{x}_j^{-1} \overline{x}_i \overline{x}_j, \qquad \overline{x}_{j+1} = \overline{x}_j.$$

There are also relations on arcs:



When an arc has an even number of bars between two crossings, it has the relations:

$$x_{i+1} = x_i, \qquad \bar{x}_{i+1} = \bar{x}_i.$$

and when there is an odd number of bars on the arc, it has the relations:

$$x_{i+1} = \bar{x}_i, \qquad \bar{x}_{i+1} = x_i.$$

Reference:

Bourgoin, Mario O. "Twisted link theory." Algebraic Geometric Topology 8.3 (2008): 1249-1279.

Kauffman, Louis H. "Virtual knot theory." Encyclopedia of Knot Theory. Chapman and Hall/CRC, 2021. 261-310.

Kim, Se-Goo. "Virtual knot groups." arXiv preprint math/9907172 (1999).