Optimizing Minimum Generating Sets of Twisted Forbidden Moves

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Mentor: Sergei Chmutov;
Main reference: *Unknotting twisted knots with Gauss diagram forbidden moves*, by Shudan Xue and Qingying Deng

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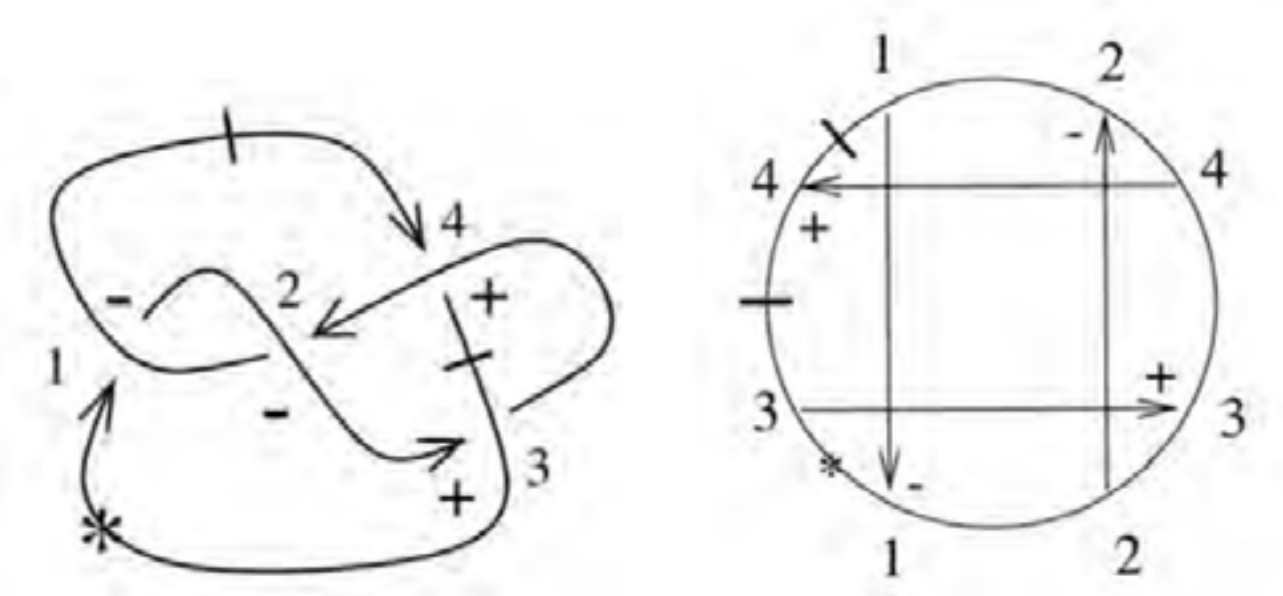
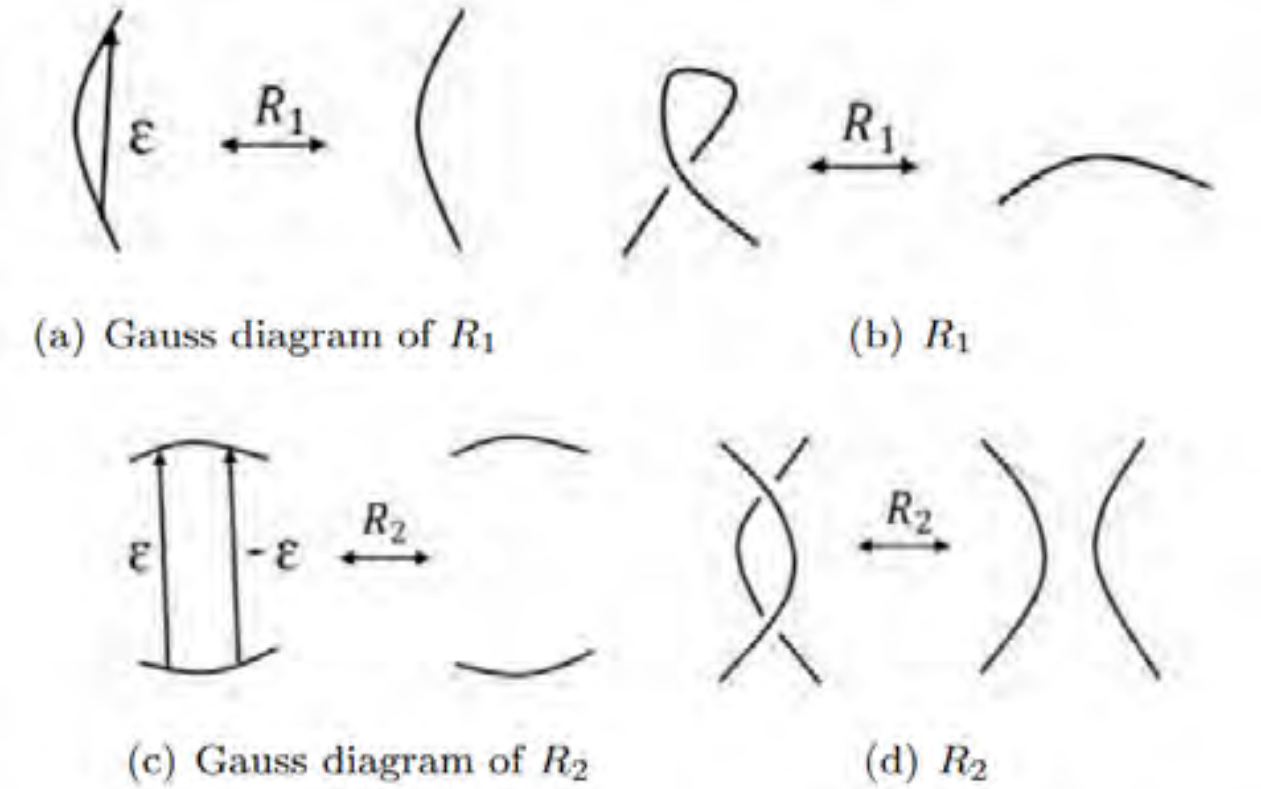
Gauss Diagrams



- Tracing the knot travels counterclockwise around the circle.
- Arrows point from the overcrossing to the undercrossing of the same numbered crossing, marked with its sign.
- Virtual crossings are not indicated on Gauss Diagrams.
- Twists are designated by bars.

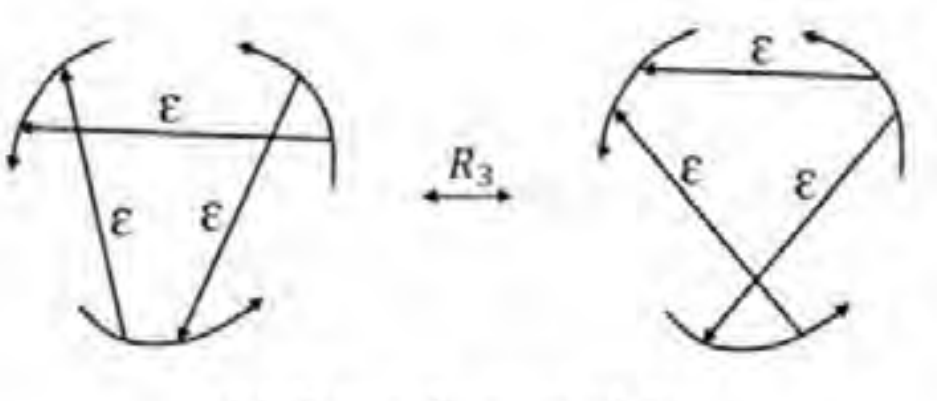
Reidemeister Moves 1&2



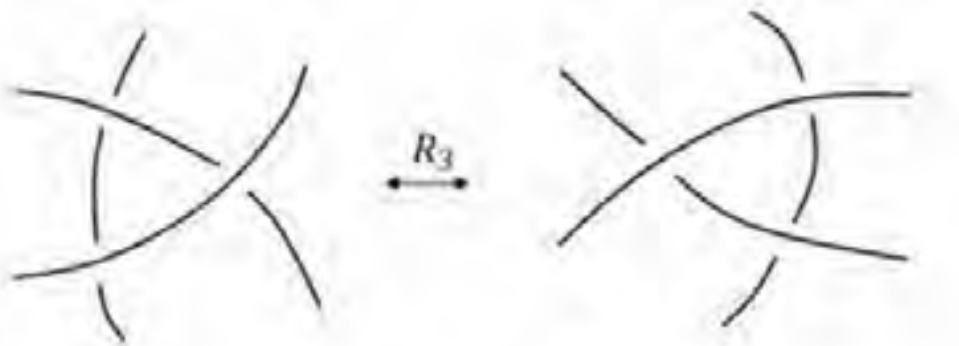
(a) Gauss diagram of R_1 (b) R_1

(c) Gauss diagram of R_2 (d) R_2

Reidemeister Move 3

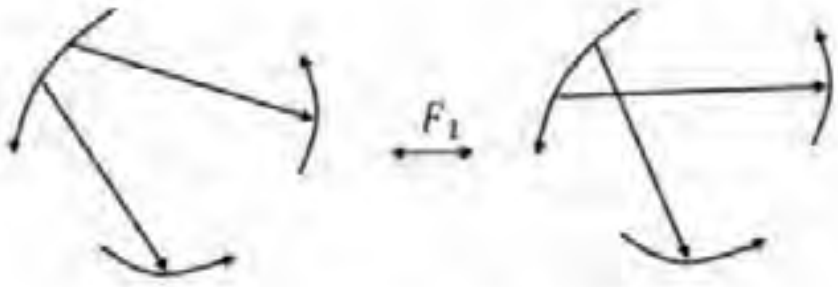


(e) Gauss diagram of R_3

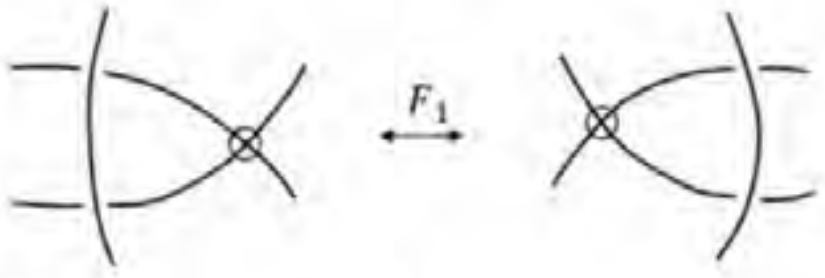


(f) R_3

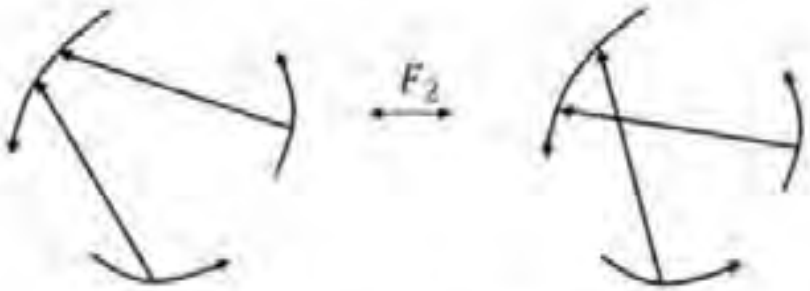
Classical Forbidden Moves



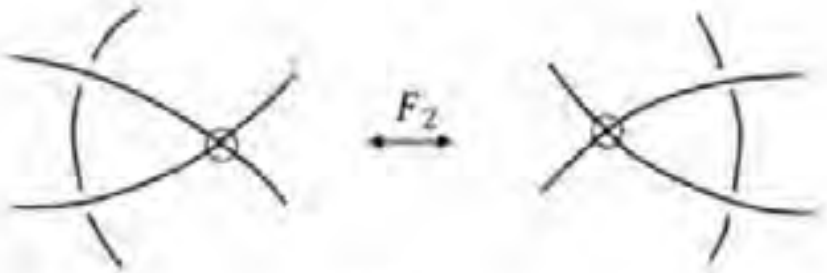
(a) Gauss diagram of F_1



(b) F_1

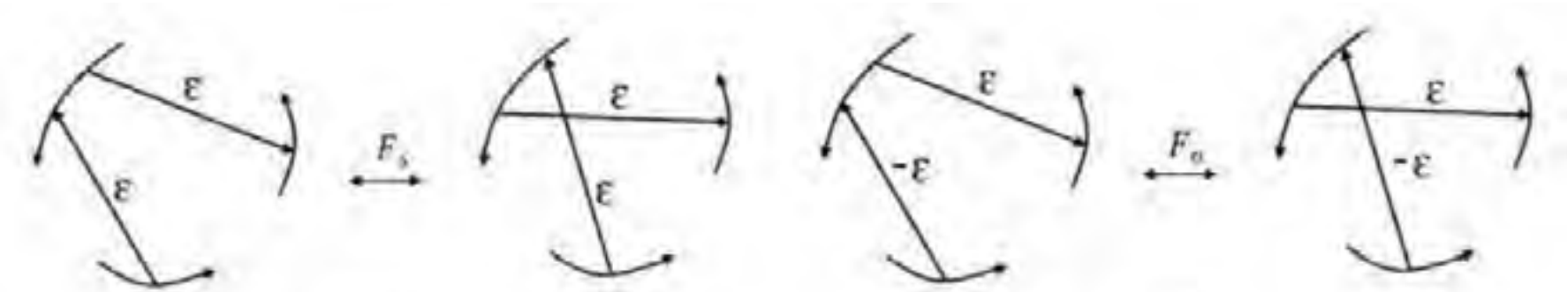


(c) Gauss diagram of F_2



(d) F_2

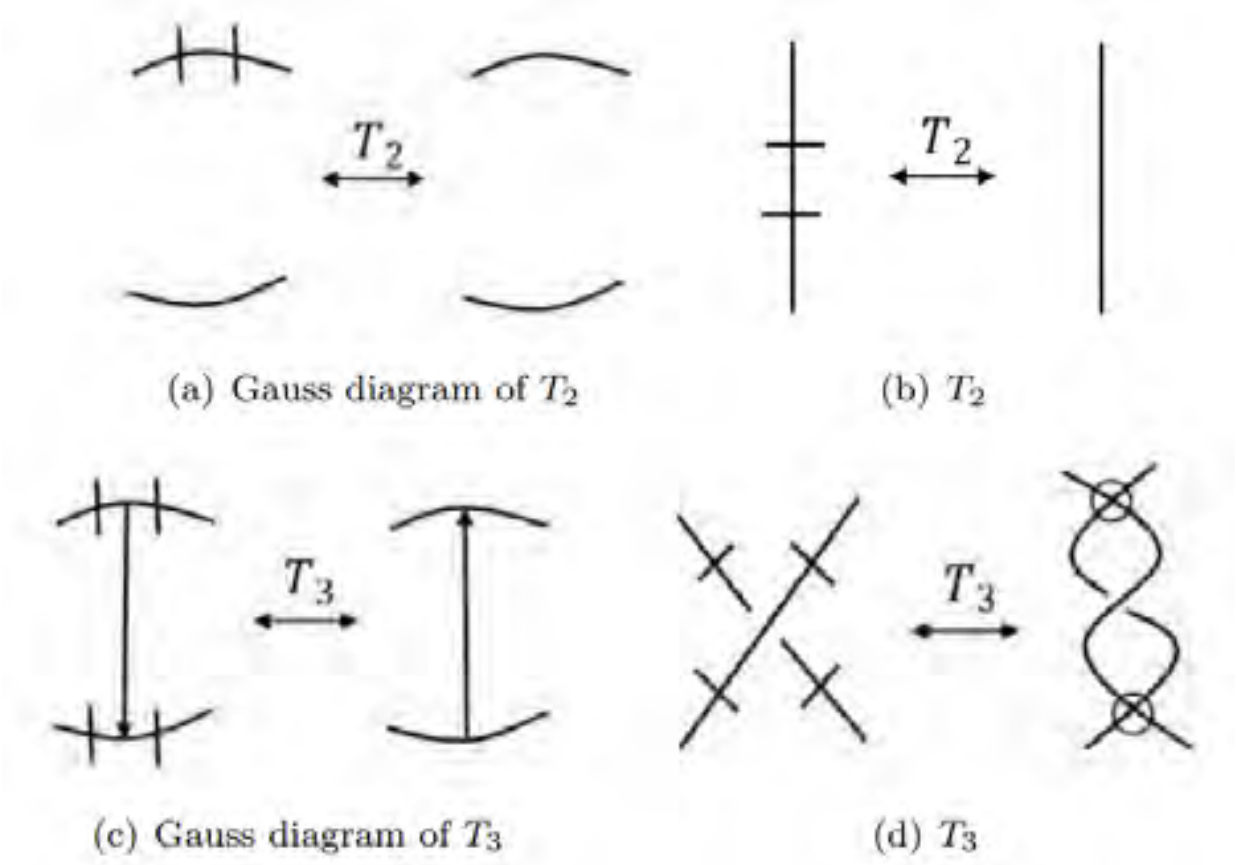
Extended Forbidden Moves



(a) F_s (b) F_o

- F_s and F_o are derivable from F_1 and F_2

Twisted Reidemeister Moves

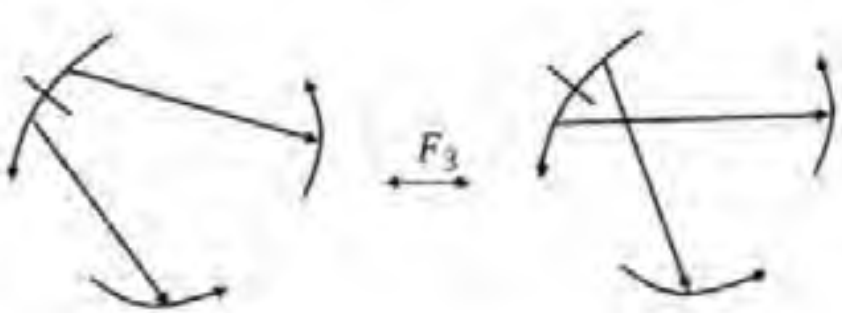


(a) Gauss diagram of T_2 (b) T_2

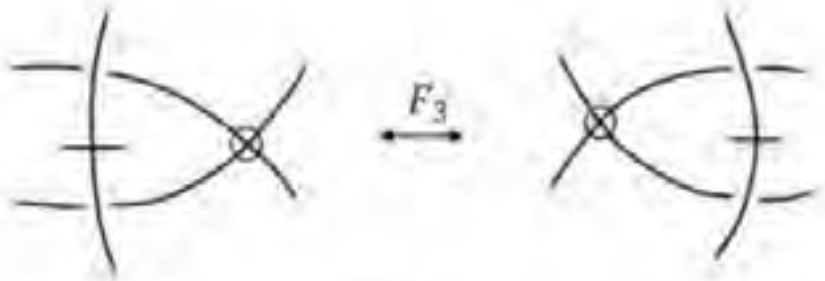
(c) Gauss diagram of T_3 (d) T_3

- T_1 moves a bar over a virtual knot, and so does not alter Gauss Diagrams

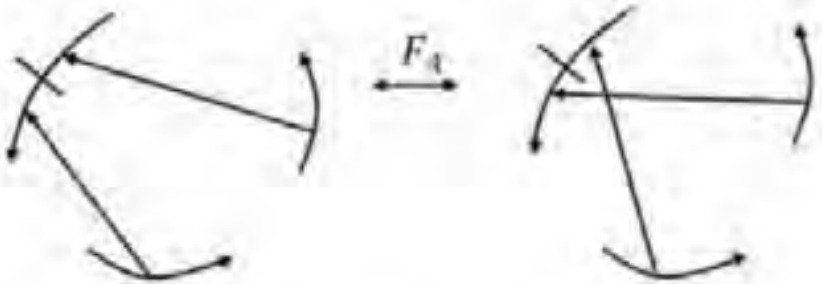
Forbidden Moves with Bars



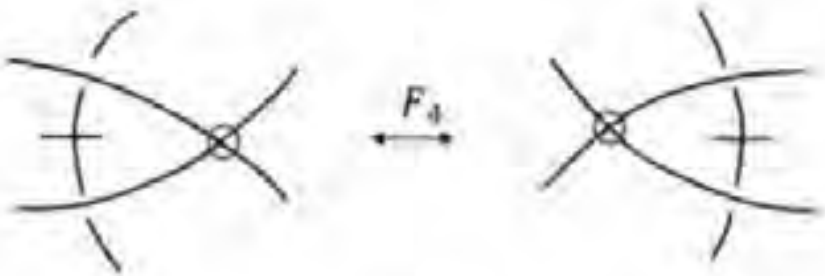
(a) Gauss diagram of F_3



(b) F_3

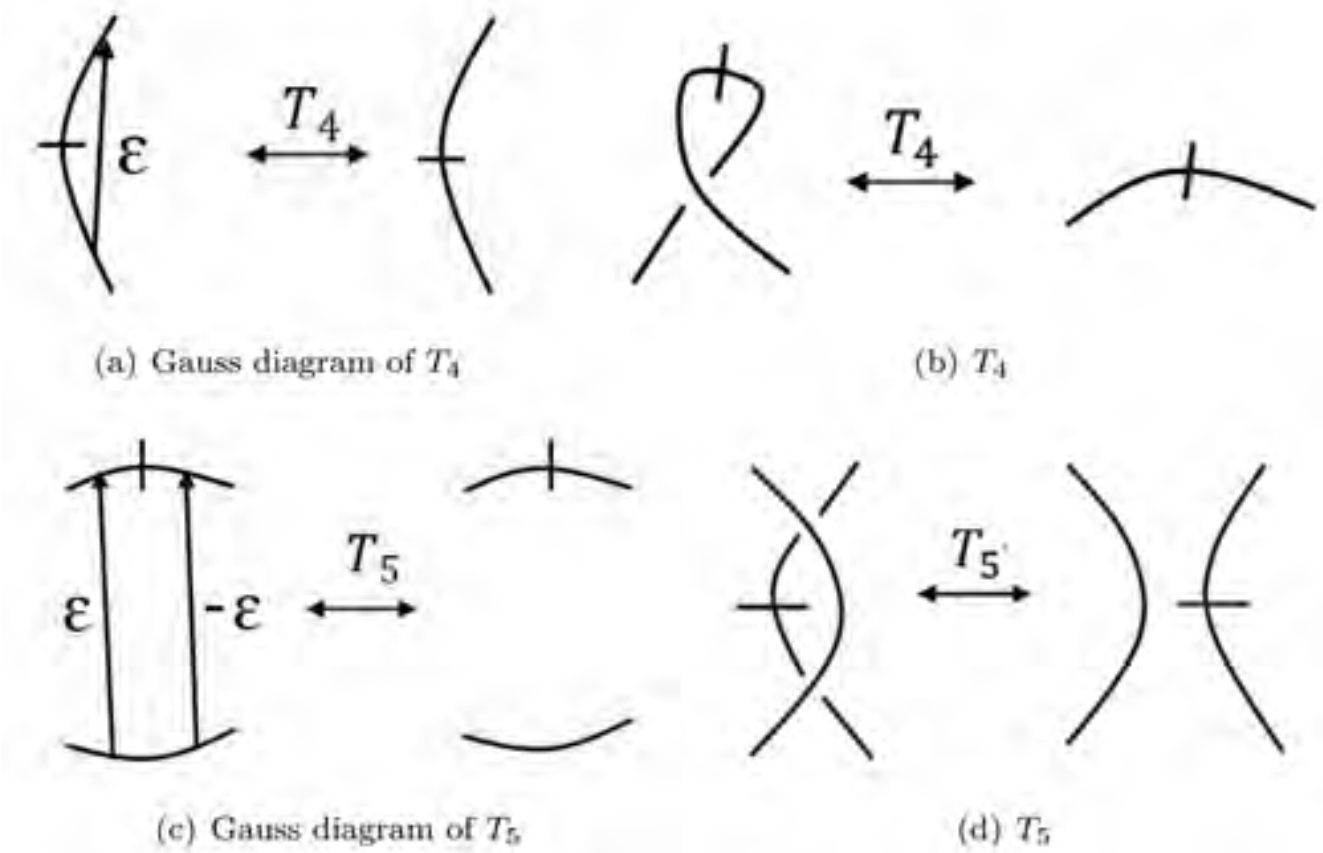


(c) Gauss diagram of F_4



(d) F_4

Reidemeister Moves 1&2 with Bars

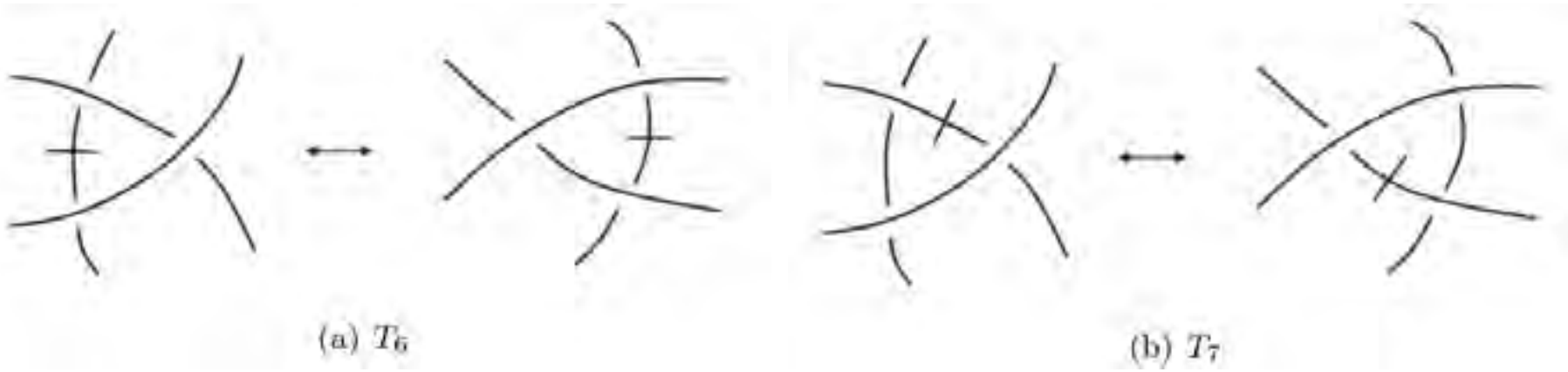


(a) Gauss diagram of T_4 (b) T_4

(c) Gauss diagram of T_5 (d) T_5

- Forbidden moves T_4 and T_5 correspond to R_1 and R_2 with bars
- T_8 represents T_5 with the bar on the overcrossing segment

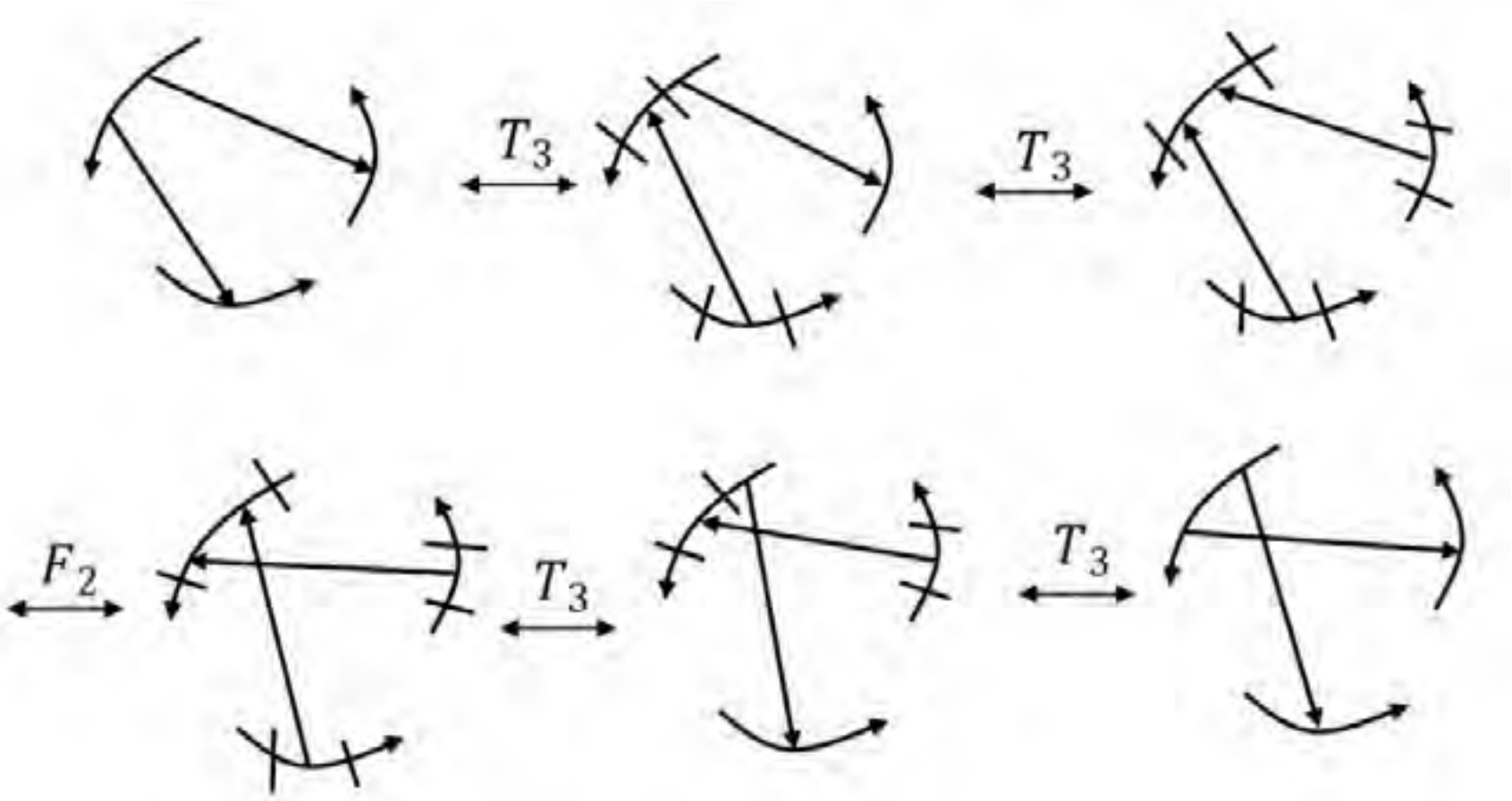
Reidemeister Move 3 with Bar



(a) T_6 (b) T_7

- Forbidden move T_6 and T_7 correspond to R_3 with a bar on different segments
- T_9 represents T_6 with the bar on an overcrossing segment

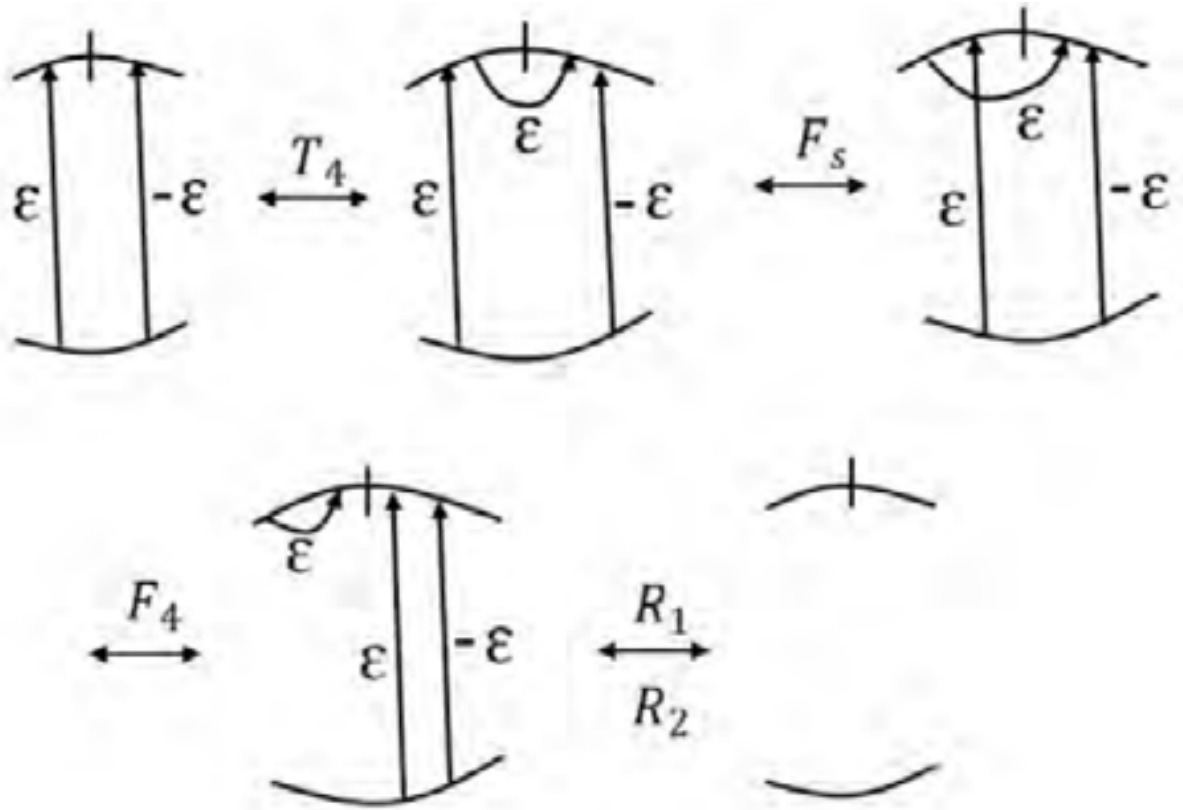
F_1 and F_2 with Twists



(a) $F_2 \to F_1$

- A similar proof shows F_3 and F_4 to be equivalent

Proof of T_5



Additional Proofs

- T_6 can be derived from F_4, F_s, and F_1
- F_u moves an arrowhead past an arrowtail of the same sign with bar. It is derived from F_1, T_4, F_s, F_3, and F_4
- F_v moves an arrowhead past an arrowtail of a different sign with bar. It is derived from F_1, T_4, F_o, F_3, and F_4
- T_7 can be derived from F_2, F_1, and F_u

Unknotting any Gauss Diagram

- Using the set of forbidden moves previously derived, any arrowhead or arrowtail can be moved past each other as well as past bars.
- All arrows can be removed by R_1 or T_4 and all pairs of bars can be removed by T_2, thus reducing all Gauss diagrams to the unknot or the unknot with a bar.
- This set of moves can be derived from T_4, F_1/F_2, and F_3/F_4