Untying Twisted Knots Using the First Reidemeister Move with Bar

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Main reference: *Unknotting twisted knots with Gauss diagram forbidden moves*,
by Shudan Xue and Qingying Deng

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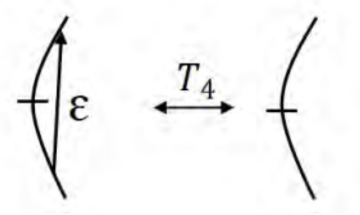
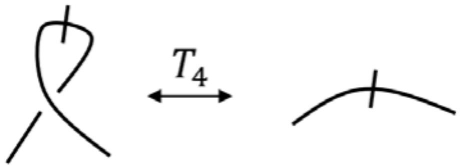
The Conjecture

Conjecture

Cardinality of minimum generating set of all forbidden moves is 3. And S contains T4 and contains one of the pairs (F1, F3), (F1, F4), (F2, F3) and (F2, F4)



(a) Gauss diagram of T_4



(b) T_4

Categorizing Moves

The moves used in the paper to untwist all knots can be grouped into two categories

Arrow Swapping Moves

- F1
- F2
- F3
- F4
- F_s
- F_o
- F_u
- F_v

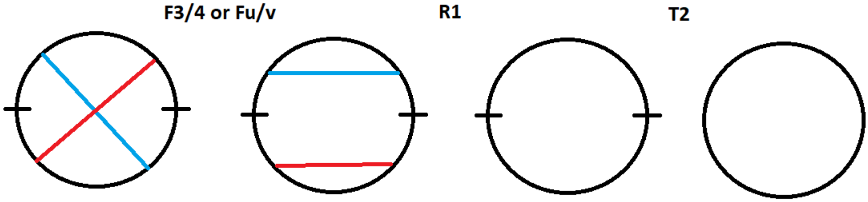
Arrow/Bar Clearing Moves

- R1
- T2
- T4

R2, R3, T3, T5, T6, T7, T8, and T9 are used as intermediate steps in proofs, but are not necessary for the paper's conclusion

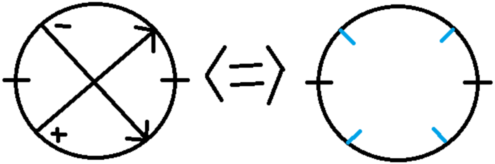
Methodology of the Paper's Proof

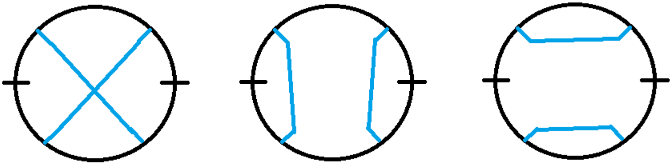
In the paper, the premise of the proof is that all Gauss Diagrams can be reduced by using the arrow swapping moves to isolate every arrow and bar, and then eliminating them using the clearing moves.



Adapting this Method

Since the arrow swapping moves cover all cases, head or tail, with or without bar, positive or negative crossing, the arrows on any Gauss Diagram can be simplified to points on the circle. Any two half-crossings could be connected by an arrow without losing information.



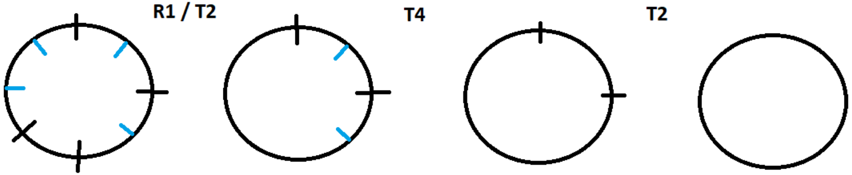


Untwisting Simplified Gauss Diagrams

The formatting of Simplified Gauss Diagrams takes for granted the arrow switching moves, which leaves the clearing moves to untie them.

- R1: Clear two adjacent half-crossings
- T2: Clear two adjacent bars
- T4: Clear two half-crossings separated by a bar

These moves can clearly reduce any Simplified Gauss Diagram. R1 and T2 first remove all adjacent pairs. This leaves an alternating string of bars and half-crossings. T4 could then remove all of the half-crossings, leaving a string of bars to be reduced by T2 to the unknot or unknot with bar.

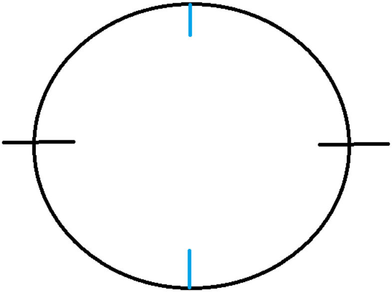


The Necessity of T4

Conjecture

There exists a Simplified Gauss Diagram which requires T4 to be reduced

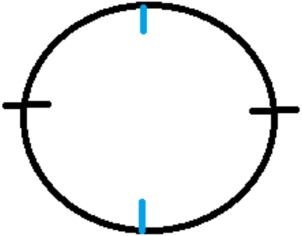
The question of this conjecture is best exemplified by attempting to reduce this diagram without T4:

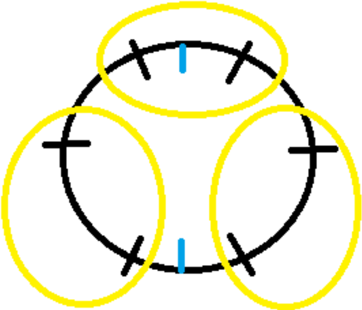


- R1 and T2 are not usable, as there are no adjacent pairs
- This leaves T3 as the only move which could alter the diagram, possibly leading to reducing it

Attempting to Use T3

In order to reduce this diagram, either the half-crossings or bars need to be grouped in pairs. Half-crossings can only be created by R1 in pairs not separated by bars. So, a reduction needs to group the bars in pairs.
T3 creates 4 bars on either side of two half-crossings. In order for these six bars to pair up, either the top or bottom two would have to be in a group, which is not possible since the two half-crossings are immovable.





Conclusion

- Therefore, given all arrow swapping moves, forbidden and otherwise, T4 is necessary to untie all Gauss Diagrams.
- Following from this, if only allowed the arrow swapping moves which were not derived from T4, T4 is still necessary to untie all Gauss Diagrams.
- This then supports the portion of the paper's conjecture that the minimum generating set contains T4.