Alternative Minimum Generating Sets of Twisted Forbidden Moves

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Main reference: *Unknotting twisted knots with Gauss diagram forbidden moves*, by Shudan Xue and Qingying Deng

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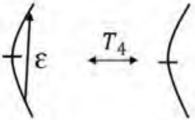
Assessing the Conjecture

Conjecture

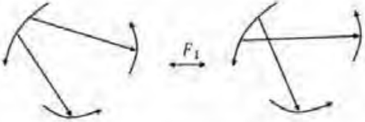
Cardinality of minimum generating set of all forbidden moves is 3. And S contains T4 and contains one of the pairs (F1, F3), (F1, F4), (F2, F3) and (F2, F4)

This is the conjecture from which our project has stemmed, and a large portion of our work has been directed at proving or disproving portions of the conjecture.

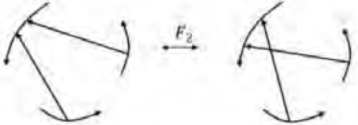
Visualized Generating Sets



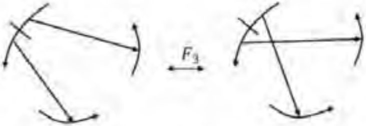
(a) Gauss diagram of T_4



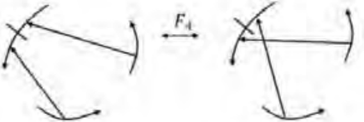
(a) Gauss diagram of F_1



(c) Gauss diagram of F_2



(a) Gauss diagram of F_3



(c) Gauss diagram of F_4

The Interpretations

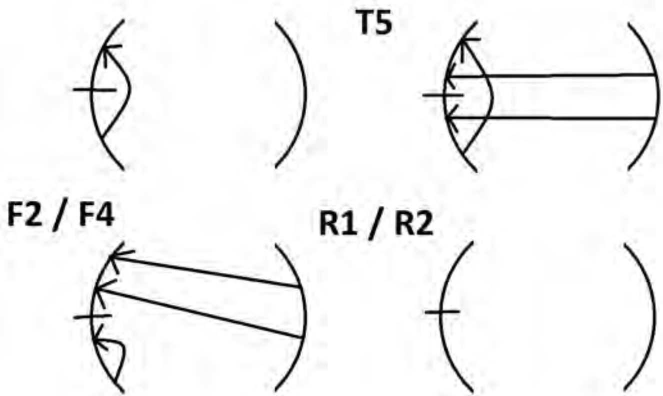
We established two interpretations of this conjecture

1. Generating sets can pull from the set of all forbidden moves:
   - The conjecture is trivially false
   - Sets of two are constructible using the "untwist"
   - The move which virtualizes a crossing defines a singleton generating set
2. Generating sets are restricted to the moves F1-4 and T4-9 as defined in the paper
   - Could be false if a set of cardinality two exists
   - or if any other moves from the restricting set can replace T4 or F1-4

In fact, there do exist substitutions not examined in the original paper

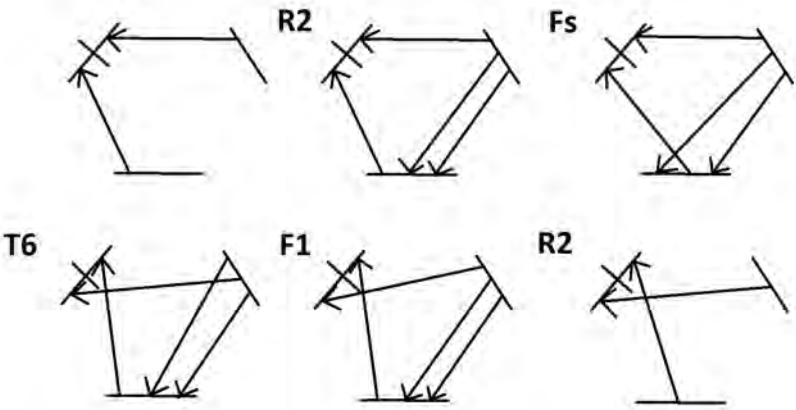
T5 - T4 Equivalence

T4 is equivalent to T5, assuming the use of the other two elements of the generating set, F1/2 and F3/4



T6 - F3/4 Equivalence

T6 is equivalent to F3/4, assuming the use of the other two elements of the generating set, T4/5 and F1/2



The New Conjecture

These substitutions being the case, a more accurate statement of the conjecture would be:

Conjecture

Cardinality of minimum generating set of all forbidden moves is 3. And S is one of the triples (T4, F1, F3), (T4, F2, F3), (T4, F1, F4), (T4, F2, F4), (T4, F1, T6), (T4, F2, T6), (T5. F1, F3), (T5, F2, F3), (T5, F1, F4), (T5, F2, F4), (T5, F1, T6), or (T5, F2, F6)

Not as clean cut a set as the original statement supposes

Remaining Unknowns

The question remains: Is the assertion of the minimum cardinality being three true?

This is the largest remaining piece of the conjecture, the rest being fairly thoroughly disproven, and it is a difficult piece to finish.

So I conclude with this conjecture, quite similar to the one which started this project

Conjecture

Let H be the set of forbidden moves $\{$F1, F2, F3, F4, T4, T5, T6, T7, T8, T9$\}$.
Let S be a subset of H such that any twisted knot can be reduced to the unknot or unknot with bar using moves in S and Reidemeister Moves.
Then, S must contain at least 3 elements, and those must include either T4 or T5; F1 or F2; and F3, F4, or T6.