Knots and Graphs

Working Group [Summer 2023]

MATH 4193, class number 16649

Instructor: ***Sergei Chmutov***

RESEARCH PROJECTS

Project 1. ***Colorings knots.*** (Gabriel Black, Mark Kikta, Charlton Li, Yan Xuan. TA: Luke Wiljanen.)

One of the simplest invariant of knots and links was the *Fox coloring*. Another is called *the Dehn coloring* which is related to the *Goeritz matrix* constructed from a knot diagram. In this project we plan to look for a generalization of Goeritz matrix concept starting with an arbitrary Kauffman state and trying to produce different coloring invariants of knots and links and relate them to the known invariants. We plan to start with papers [HIMY] and [Kol]. The other relevant papers to this project are [CSW] and [Tra]. A general standard excellent introduction to knot theory is [Ad].

Project 2. ***Arrow polynomial.*** (Jeremy Case, Pranav Jois, Leon Lozinskiy, Yumin Shen. TA: Deniz Genlik)

The Jones polynomial of links can be generalized to more powerful HOMFLYPT polynomial. From the other hand an extension of the Jones polynomial to virtual links of L.Kauffnam [Ka] can be generalized to so called *Arrow polynomial* [DK, BBC]. One goal of this project is try to extend HOMFLYPT polynomial to virtual links and find its arrow version. We would like to adopt the expression for the HOMFLYPT polynomial from [ChPo] to virtual links and then try to introduce arrow structure on the corresponding states.

Another application of the arrow polynomial goes like follows. The regions of a link diagrams always can be colored in a checkerboard manner. This is no longer true for virtual links. One obstacle for checkerboard colorability comes from the arrow polynomial [DJK]. It solves the colorability problems for all virtual knots up to 4 crossings with one exception, 4.72, for which the colorability is still open. Another goal of this project is to simply the proof from [DJK], look for other possible obstacles, and resolve the question about 4.72.

References

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[Tra] L. Traldi,*Link colorings and the Goeritz matrix*, J. Knot Theory Ramifications **26**(8) (2017) #1750045 (19 pages).