Recursion for the Partial-Dual Euler Genus Polynomial

Charlton Li (li.12635@osu.edu)
The Ohio State University [Mentor:Sergei Chmutov]

Abstract of Report Talk: In topological graph theory, ribbon graphs are surfaces with boundary made up of vertex-discs and edge-ribbons and are equivalent to graphs cellularly embedded in a surface. Geometric duality is an operation that swaps the roles of the vertices and faces of a ribbon graph. Chmutov defined partial duality as the application of geometric duality to a spanning ribbon subgraph. Gross, Mansour, and Tucker introduced the partial-dual Euler genus polynomial, a generating function that counts partial duals of a ribbon graph by their Euler genus.

Recently, Yan and Jin showed that if two bouquets (one-vertex ribbon graphs) have the same signed intersection graph, then they have the same partial-dual polynomial. Thus the intersection polynomial of a signed intersection graph G can be defined as the partial-dual polynomial of any bouquet whose signed intersection graph is isomorphic to G.

We derive a recurrence relation for the intersection polynomial at cut-vertices, generalizing a recursion by Yan and Jin that applies to leaves. Incidentally, the twist polynomial defined by Yan and Jin for delta-matroids defines a polynomial on the class of simple signed graphs that is identical to the intersection polynomial when restricted to signed intersection graphs. Our recurrence relation extends to this more general setting. An interesting direction for future work is to find a completely recursive definition for the intersection polynomial or twist polynomial on simple signed graphs (analogous to the deletion-contraction relations of the Tutte polynomial).