Multivirtual Knot Theory

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Definition

A multivirutal knot diagram is a virtual knot diagram in which each virtual crossing is assigned a "type".



Definition

A multivirtual knot is an equivalence class of multivirtual knot diagrams under the classical and multivirtual Reidemeister moves as well as planar isotopy.

Multivirtual Reidemeister Moves



Multivirtual Reidemeister Moves



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Multivirtual Reidemeister Moves



The multivirtual Reidemeister moves are equivalent to the multivirutal detour moves where a series of consecutive virtual crossings is removed from the diagram and replaced with an arc with the same endpoints with any crossings on this new arc being virtual crossings of the same type.



The chromatic bracket of a multivirtual knot diagram K is equal to the following sum over all splittings of the diagram. $\langle K \rangle = \sum_{S} A^{a(s)} B^{bs} \langle S \rangle$ Where a(s) is the number of a splittings, b(s) is the number of b splittings and $\langle S \rangle$ is the state of the diagram after the splittings. To evaluate the state of S we use the following rules

$$\langle O \rangle = \delta$$
$$\langle X \rangle = 2 \langle X \rangle - \langle X \rangle$$

Where this dot represents a new type of nodal crossing.

Multivirtual Bracket in the Case of 2 Crossing Types

contract delete to , X to me , X to ... ↔ ····= 2· -·· = 28-82 ↔ = 20 - - = 2(2:) - 25+5² =28-28+82=+8200 $\leftrightarrow \nabla = 2\beta - \sqrt{-2\beta^2 - 2/+1}$ = $2\beta^2 + (-2+\beta)(2\beta - \beta^2)$ =-83+682-45.

With the substitution of $B = A^{-1}$ and $\delta = -A^2 - A^{-2}$ the chromatic bracket is invariant under all multivirtual and classical Reidemeister moves except for R1.

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Multi-virtual knot theory.

Journal of Knot Theory and it's Ramifications, 2025.

Louis H. Kauffman and Sujoy Mukherjee and Petr Vojtěchovský Algebraic invariants of multi-virtual links. arXiv, 2025.