Knots and Graphs

Undergraduate Research Working Group [Summer 2025]

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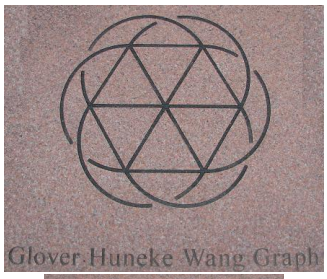
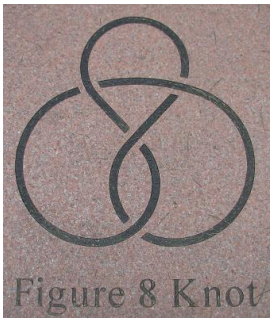
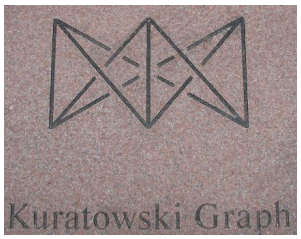
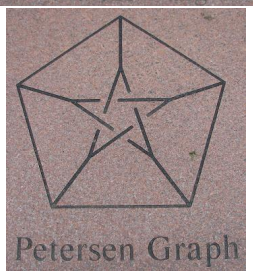
ANNOUNCEMENT

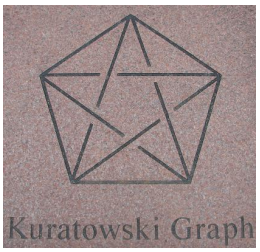
Everyone entering Math Tower put his foot on these Knots and Graphs.





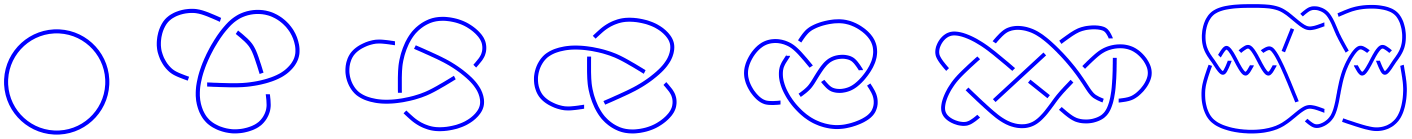




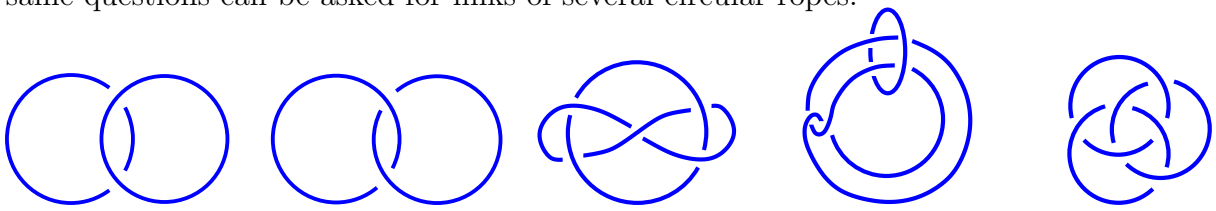


We are going to put our hands on them.

Imagine a rope tied somehow and have ends glued together afterward, forming a circular tied rope. Having two such ropes, can you move one of them around to get the other one? In particular, can you untie it and get a perfect circle? Take a look at the ropes below and think which of them you can untie, which you can not, for which two of them you can move one around and get the other.

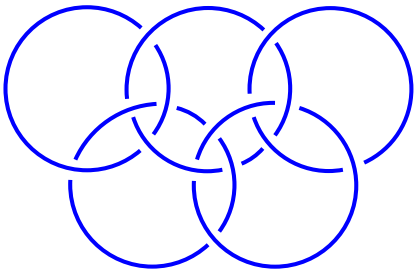


The same questions can be asked for links of several circular ropes:



The last link is known as "Borromean rings"'. It has a nice property that if any component is removed then the other two will fall apart.

Similarly, for the Olympic rings



there is a circle whose removing leads the remaining four to fall apart. Which one?

One way to study such questions mathematically is to assign some algebraic object, for instance, a polynomial (or a group, if you have already taken abstract algebra), to a picture of knotted rope in an invariant way. That is, if we move the rope around and get a different picture, then the objects corresponding to the two pictures must be the same (polynomials are equal, groups are isomorphic etc.).

During the program we are going to study such invariants and relate them to combinatorial graphs. Knots and graphs play a very important role in physics (quantum field theory) where they appear as Feynman diagrams, in biology where knots and graphs model protein structures, in computer science where they represent certain types of networks, and in many other areas.

During the first two weeks I am planning give an introduction and a necessary theoretical background and state the problems for small projects. These are going to be in person. Then we will switch entirely online and the students will do presentations on their projects.

The whole group will meet twice a week (Monday, Friday 3:00-4:00pm) to discuss general theory, techniques, and progress in our small projects. Each project will contain some open problems whose solutions are unknown so far. I expect a small group of about 3-4 students working on each project. I plan to meet (on ZOOM) with each small research group about one more time per week. On these small meetings we will discuss specific problems, methods, solutions related to the specific projects. I will announce all zoom meetings in Carmen for everyone, so anybody can join us for smaller meetings if you are interested and willing to even if it is not your project. So, all together, you are supposed to have 3 one-hour meetings per week with me. And perhaps more withing your small group.

We are going to have three mentors from our graduate students. You are welcome to chat with them, they will help you to understand everything you need for your projects, tell you about graduate schools and their programs etc.