Knots and Graphs

Working Group [Summer 2026]

MATH 4193, class number 15031

Instructor: ***Sergei Chmutov***

RESEARCH PROJECTS

Project 1. ***The Penrose polynomial.*** (Max Brown, Ethan Lu, Minxiao Wu. TA: Luke Wiljanen.)

Originally the *Penrose polynomial* is defined for plane cubic graphs with a perfect matching. It is responsible for the colorings of the graphs. There is a generalization of it to graphs on surfaces (ribbon graphs). We are going to study this polynomial, its properties, and its applications to the knot theory. In particular we would like to compare the generalizations of the Penrose polynomial to ribbon graphs from [KSW] and from [EM]. The further development of this project may go to categorifications of the Penrose and related polynomials [Ka] and their generalizations to matroids and delta-matroids [EM].

Project 2. ***Twist polynomial of Delta-matroids.*** (Yile Huang, Logan Keck, Ryland Smoot, David Dumanskyy. TA: Luci Krnic)

A *delta-matroid* is a combinatorial structure capturing many properties of ribbon graphs which are graphs cellularly embedded into a surface. C.Chun, I.Moffatt, S.Noble, and R.Rueckriemen introduced [CMNR] a concept of *width* of a delta-matroid. This notion generalizes the genus of the surface of a ribbon graph. The correspondent generating function over all twists of a delta matroid is called the *twist polynomial*. In the case of ribbon graphs it is called the *partial dual genus polynomial*.

In this project we plan to study this polynomial and its relations to other delta-matroids and ribbon graphs polynomials. We plan to start with the papers of X.Jin, Z.Li, Q.Yan, G.Zhang, *On the maximum twist width of delta-matroids* [JLYZ] and of G.Zhang, Q.Yan, *Twist polynomial interpolation for binary delta-matroids* [ZY] and try to generalize their results and relate them to other polynomials.

References

[CMNR] C. Chun, I. Moffatt, S. Noble, R. Rueckriemen, *Matroids, delta-matroids and embedded graphs*, Journal of Combinatorial Theory,Ser. A **167** (2019) 7–59.

[EM] J. A. Ellis-Monaghan, I. Moffatt, *Graphs on Surfaces: Dualities, Polynomials, and Knots*, Springer, 2013.

[Ka] L. Kauffman, *Categorification of Chromatic, Dichromatic and Penrose Polynomials*, Preprint `arXiv:2512.21027v1 [math.CO]` .

[KSW] L. Kauffman, D. Silver,S. Williams *The Penrose-Kauffman Polynomial*, Preprint `arXiv:2604.16635v1 [math.GT]`.

[JLYZ] X. Jin, Z. Li, Q. Yan, G. Zhang, *On the maximum twist width of delta-matroids*, Preprint `arXiv:2602.01946v1 [math.CO]`.

[ZY] G. Zhang, Q. Yan, *Twist polynomial interpolation for binary delta-matroids*, Preprint `arXiv:2605.05601v1 [math.CO]`.