Ohio State University

Practice Midterm - Feb 26, 2013

Winter 2012, MATH 5602

Instructor: Ching-Shan Chou

Closed book examination Time: 3:00–3:55pm

Last Name: ____________________, **First:** _____________ **Signature** ____________________

Student Number ______________________

Special Instructions:

- Be sure that this examination has 10 pages. Write your name on top of each page.
- To get full credit, you have to present your arguments/calculations in a clear fashion, together with correct reasoning. It is important to include all details.
- If you want to use some results we have discussed before, be sure to clearly state the results you use.
- Only pens (or pencils + erasers) are allowed during the exam. If you need scratch papers, use the back.
- No calculators of any kind will be allowed.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

1		20
2		40
3		20
Total		100

1. Consider the heat equation

$$u_t = u_{xx},$$

and a finite difference method for this equation

$$\frac{u_i^{n+1} - u_i^{n-1}}{2\Delta t} - \frac{u_{i+1}^n - 2\{\theta u_i^{n+1} + (1-\theta)u_i^{n-1}\} + u_{i-1}^n}{(\Delta x)^2} = 0,$$

where $0 \leq \theta \leq 1$.

(a) What is the local truncation error of this method?

(b) Discuss the consistency of this method when (i) $\Delta t = r\Delta x$ and (ii) $\Delta t = r(\Delta x)^2$.

1. (continued)

2. Consider the heat equation

$$u_t = u_{xx}, \quad 0 \le x \le 1, \quad 0 \le t < \infty,$$

with no-flux boundary conditions ($u_x(0,t) = u_x(1,t) = 0$) for all $t > 0$, and initial condition $u(x,0) = \cos(2\pi x) + 1$.

(a) When t goes to ∞, the solution goes to a steady state. Describe the steady state function (e.g. linear, quadratic....).

(b) Show that $\int_0^1 u(x,t)\,dx$ is a constant.

(c) Implement the simplest explicit method (forward Euler in time, second order central difference in space), and run until $T = 1$. Show the figures of $t = 0, 0.25, 0.5, 0.75, 1$ on the same plot. (d) Let's change the boundary condition to periodic boundary condition, show that $\int_0^1 u(x,t)\,dx$ is still a constant.

(e) With period boundary conditions, we can perform Von Neumann stability analysis. Show that the scheme used in (c) is stable under certain constraint of $\Delta t/(\Delta x)^2$ (write out the constraint too).

2. (continued)

3. Consider the transport equation

$$u_t + u_x = 0, \quad 0 \le x \le 1, \quad 0 \le t < \infty.$$

(a) Write down the upwind method and Lax-Friedrichs method.
(b) Perform local truncation error analysis for both method. Compare these method and explain why the upwind method is more dissipative than the Lax-Friedrichs method. (hint: the dissipation, or called diffusion is what contributes to the smearing around the discontinuities).

3. (continued)