

Mathematical Induction

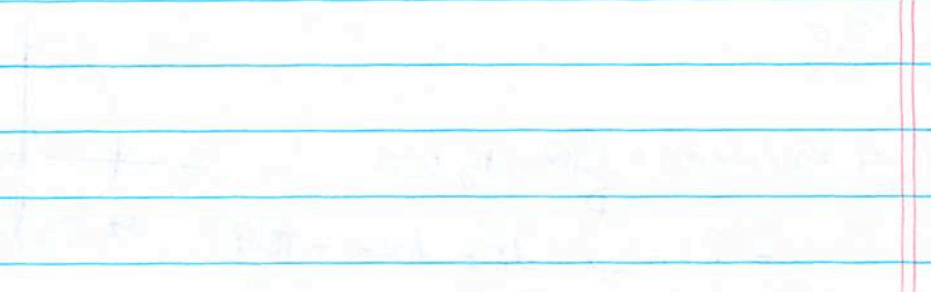
1. Base Case: $n=1$

2. Inductive Hypothesis: Assume true for $n=k$

3. Inductive Step: Prove true for $n=k+1$

4. Conclusion: True for all $n \in \mathbb{N}$

5. Example: $1+2+\dots+n = \frac{n(n+1)}{2}$



6. Example: $2^n > n^2$ for $n \geq 5$

7. Example: $1^3 + 2^3 + \dots + n^3 = (\frac{n(n+1)}{2})^2$

8. Example: $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

9. Example: $1^4 + 2^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

10. Example: $1^5 + 2^5 + \dots + n^5 = \frac{n^2(n+1)^2(2n^2+5n+3)}{12}$

11. Example: $1^6 + 2^6 + \dots + n^6 = \frac{n(n+1)(2n+1)(3n^3+6n^2-3n-1)}{42}$

12. Example: $1^7 + 2^7 + \dots + n^7 = \frac{n^2(n+1)^2(2n^2+5n+3)(3n^2+3n-1)}{24}$

13. Example: $1^8 + 2^8 + \dots + n^8 = \frac{n(n+1)(2n+1)(3n^3+6n^2-3n-1)(7n^2+7n-1)}{90}$

14. Example: $1^9 + 2^9 + \dots + n^9 = \frac{n^2(n+1)^2(2n^2+5n+3)(3n^2+3n-1)(5n^2+5n-3)}{252}$

15. Example: $1^{10} + 2^{10} + \dots + n^{10} = \frac{n(n+1)(2n+1)(3n^3+6n^2-3n-1)(5n^2+5n-3)(7n^2+7n-1)}{66}$

FINAL REVIEW

① Calculate $\oint_D (y dx + 2x dy)$ where C

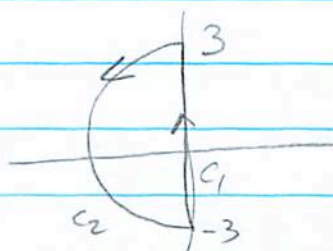
is the left half circle $x^2 + y^2 = 9$ followed by the segment joining $(0, -3)$ and $(0, 3)$ oriented counter-clockwise in 2 ways

a) Using Green's theorem

b) Calculating directly the line integral

Sol.

$$\begin{aligned} \text{a) } \oint_D P dx + Q dy &= \iint_D (Q_x - P_y) dA \\ &= \iint_D (2 - 1) dA = \text{Area} = \frac{\pi \cdot 9}{2} \end{aligned}$$



$$\begin{aligned} \text{b) } \oint_D &= \int_{C_1} + \int_{C_2} & \text{On } C_1: & \begin{cases} x=0 \\ y=t \end{cases} \text{ from } -3 \text{ to } 3 \\ & & & \frac{dx}{dt} = 0, \frac{dy}{dt} = 1 \end{aligned}$$

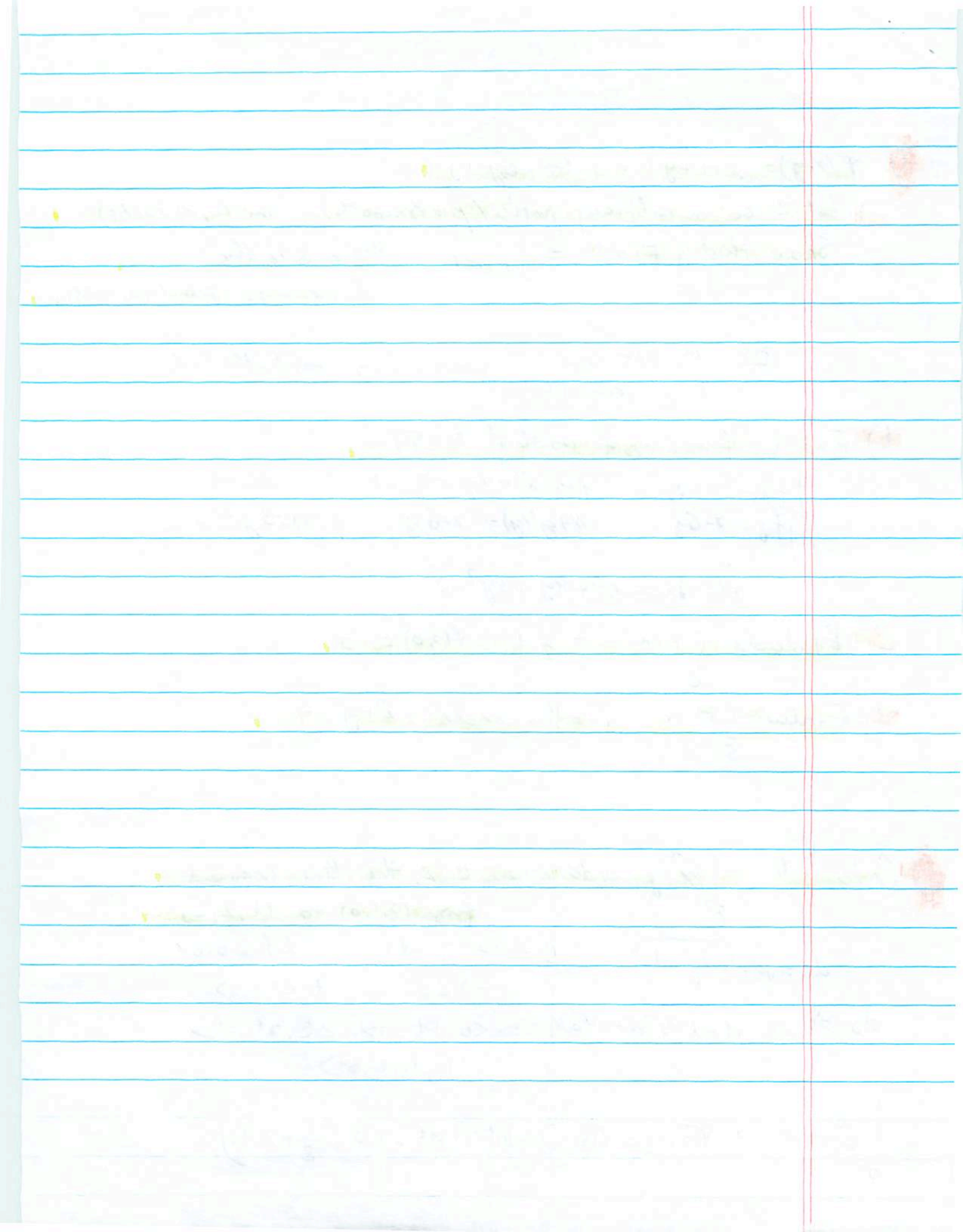
$$\int_{C_1} = \int_{-3}^3 (t \cdot 0 + 2 \cdot 0 \cdot 1) dt = 0$$

$$\int_{C_2} = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (3 \sin t \cdot (-3 \cos t) + 6 \cos t \cdot 3 \cos t) dt$$

$x = 3 \cos t$
 $y = 3 \sin t$ t from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-9 \sin^2 t + 18 \cos^2 t) dt = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -9 \frac{1 - \cos 2t}{2} + 18 \frac{1 + \cos 2t}{2} dt = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(\frac{9}{2} + \left(\frac{9}{2} + 18 \right) \cos 2t \right) dt$$

$$= \frac{9}{2} \pi \quad \checkmark$$



① $\underline{F(x,y)} = (2x+y)\underline{i} + (x+6y)\underline{j}$

a) Let C be an arbitrary path (piecewise smooth) from $(0,0)$ to $(1,1)$

Show that $\int_C \underline{F} \cdot d\underline{r} = \int_S \underline{F} \cdot d\underline{r}$ where S is the line segment from $(0,0)$ to $(1,1)$

Sol. $P_y = 1 = Q_x$ \Rightarrow conservative \Rightarrow path independent
 Dom - simply connected

b) Find the functions f so that $\underline{F} = \nabla f$

$$\begin{cases} f_x = 2x+y & f = x^2 + xy + g(y) \\ f_y = x+6y & x + g'(y) = x+6y \quad g(y) = 3y^2 + C \end{cases}$$

$$f(x,y) = x^2 + xy + 3y^2 + C$$

c) Evaluate $\int_C \underline{F} \cdot d\underline{r} = f(1,1) - f(0,0) = 3$

d) Evaluate $\int_S \underline{F} \cdot d\underline{r}$; $S =$ the circle $x^2 + y^2 = 4$

② Calculate $\int_C (x+y+z) ds$ where C is the line segment from $(0,1,0)$ to $(1,2,3)$

$$ds = \sqrt{dx^2 + dy^2 + dz^2} \quad \underline{r} = (1-t)\underline{r}_0 + t\underline{r}_1 \quad t \text{ from } 0 \text{ to } 1$$

$$ds = \frac{ds}{dt} dt = \sqrt{1+1+9} dt = \sqrt{11} dt$$

$$\underline{r} = (1-t)\langle 0,1,0 \rangle + t\langle 1,2,3 \rangle = \langle 0,1-t,0 \rangle + \langle t,2t,3t \rangle = \langle t,1+t,3t \rangle$$

$$\int_0^1 (t^2 + 1 + t + 3t) \sqrt{11} dt = \sqrt{11} \int_0^1 (t^2 + 4t + 1) dt = \sqrt{11} \left(\frac{1}{3} + 2 + 1 \right)$$

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(3) Use spherical coord to calculate $\iiint_E x \, dV$

where E is the solid between the spheres of radius 1 and 2 above the cone $z = \sqrt{x^2 + y^2}$

$$1 \leq \rho \leq 2$$

$$z \geq \sqrt{x^2 + y^2} \quad \rho \cos \varphi \geq \rho \sin \varphi \quad \cos \varphi \geq \sin \varphi$$

$$0 \leq \varphi \leq \frac{\pi}{4}$$

$$2\pi/4 \quad 2\pi$$

$$\int_0^{2\pi/4} \int_0^{2\pi} \int_1^2 \rho \sin \varphi \cos \theta \rho^2 \sin \varphi \, d\theta \, d\rho \, d\varphi$$

$$= \frac{1}{4} \rho^4 \Big|_1^2 \cdot 2\pi \cdot \int_0^{\pi/4} \sin^2 \varphi \cos \varphi \, d\varphi = \frac{1}{4} (2^4 - 1) \cdot 2\pi \cdot \frac{1}{3} \sin^3 \varphi \Big|_0^{\pi/4}$$



(4) Calc. $\iiint_E x \, dV$, $E =$ inside the sphere rad 1, in the octant $x > 0, y > 0, z < 0$

$$0 \leq \rho \leq 1$$

$$0 \leq \theta \leq \pi/2$$

$$\frac{\pi}{2} \leq \varphi \leq \pi$$



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⑤ Use a \iiint to calc the vol of the solid bd by the cylinder $y = 3 - x^2$ and the planes

$$x=0, y=0, z=0, 2x+y+z=4$$

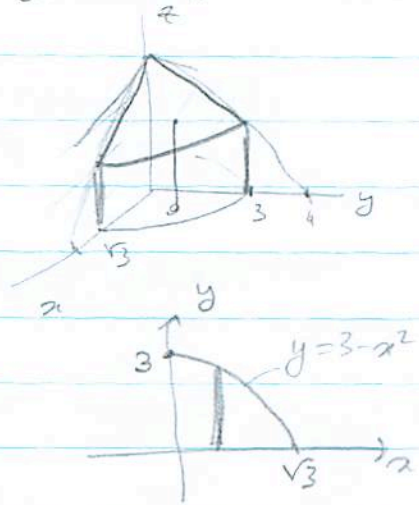
$$V = \iiint_E dV = \iint \int_0^{4-2x-y} dz \, dA$$

$$= \int_0^{\sqrt{3}} \int_0^{3-x^2} \int_0^{4-2x-y} dz \, dy \, dx$$

$$= \int_0^{\sqrt{3}} \int_0^{3-x^2} (4-2x-y) \, dy \, dx$$

$$= \int_0^{\sqrt{3}} \left. 4y - 2xy - \frac{1}{2}y^2 \right|_{y=0}^{y=3-x^2} dx$$

$$= \int_0^{\sqrt{3}} \left(4(3-x^2) - 2x(3-x^2) - \frac{1}{2}(3-x^2)^2 \right) dx = \dots$$



1. The first part of the paper is devoted to a general discussion of the problem.

2. In the second part, we consider the case of a homogeneous medium.

3. The third part is devoted to the case of an inhomogeneous medium.

4. In the fourth part, we consider the case of a medium with a periodic structure.

5. The fifth part is devoted to the case of a medium with a random structure.

6. In the sixth part, we consider the case of a medium with a periodic structure and a random structure.

7. The seventh part is devoted to the case of a medium with a periodic structure and a random structure.

8. In the eighth part, we consider the case of a medium with a periodic structure and a random structure.

9. The ninth part is devoted to the case of a medium with a periodic structure and a random structure.

10. In the tenth part, we consider the case of a medium with a periodic structure and a random structure.

11. The eleventh part is devoted to the case of a medium with a periodic structure and a random structure.

12. In the twelfth part, we consider the case of a medium with a periodic structure and a random structure.

13. The thirteenth part is devoted to the case of a medium with a periodic structure and a random structure.

14. In the fourteenth part, we consider the case of a medium with a periodic structure and a random structure.

⑥ $f(x, y) = x^2 + y^2 - 4y$

a) Find the critical pts and use the 2nd deriv test

$$f_x = 2x = 0 \quad (0, 2)$$

$$f_y = 2y - 4 = 0$$

$$f_{xx} = 2, f_{yy} = 2, f_{xy} = 0 \quad \Delta = 4 > 0, f_{xx} > 0 \text{ min}$$

b) Use the Lagr mult to find the extrema on the ellipse $x^2 + 2y^2 = 9$

$$\begin{cases} 2x = 2\lambda x \\ 2y - 4 = 2\lambda y \\ x^2 + 2y^2 = 9 \end{cases} \quad \begin{cases} x(\lambda - 1) = 0 \\ 2y - 4 = 2\lambda y \\ x^2 + 2y^2 = 9 \end{cases} \quad \begin{array}{l} \textcircled{I} \lambda = 0 \Rightarrow y = \pm 3 \\ \textcircled{II} \lambda \neq 0 \Rightarrow \lambda = 1 \Rightarrow 2y - 4 = 2y \\ \quad \quad \quad 2y = 4 \\ \quad \quad \quad y = 2 \end{array}$$

$$x^2 + 2 \cdot 4 = 9$$

$$x = \pm 1 \quad (\pm 1, 2)$$

$$f(0, \sqrt{3}) = 3 - 4\sqrt{3}$$

$$f(1, 2) = 1 + 4 - 8 = -3 = f(-1, 2)$$

$$f(0, -\sqrt{3}) = 3 + 4\sqrt{3} \quad \leftarrow \text{Max}$$

$$3 - 4\sqrt{3} \approx 3 - 4 \cdot 1.71 = 3 - 6.84 = -3.84 \quad \leftarrow \text{min}$$

c) Find the extrema of f on the elliptical disk $x^2 + 2y^2 \leq 9$ using a, b. Explain how these results are useful.

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a) What is the domain of \ln ↘

7) b) Write the linear approx of $f(x, y, z) = x \ln(2x + 3y + 4z)$ at the point $(1, 1, -1)$

$$f(1, 1, -1) = 1 \ln(2 + 3 - 4) = 1 \ln 1 = 0$$

8) Show that $u(x, y) = f(x - 2y)$ with f twice diff with f'' cont

satisfies $4u_{xx} - u_{yy} = 0$

$$u_x = f'(x - 2y)$$

$$u_{xx} = f''(x - 2y)$$

$$u_y = -2 f'(x - 2y)$$

$$u_{yy} = 4 f''(x - 2y)$$