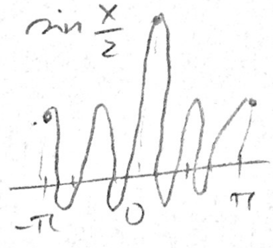
We saw that:

Dirichlet kernel $D_N(x) = \sum_{-N}^{N} e^{inx} = \dfrac{\sin(N+\frac{1}{2})x}{\sin\frac{x}{2}}$



– huge max at $x=0$, $=2N+1$

– many zeros at $\frac{2k\pi}{2N+1}$

Useful because

$$S_N(f) = \sum_{-N}^{N} \hat{f}_n e^{inx}$$

$$S_N f(x) = \frac{1}{2\pi} f * D_N .$$

Theorem $(\widehat{f*g})_n = \hat{f}_n \hat{g}_n \cdot 2\pi$

Also Theorem $\boxed{(\widehat{fg})_n = (\hat{f} * \hat{g})_n := \sum_k \hat{f}_k \hat{g}_{n-k} \text{ discrete convolution.}}$

Indeed $\widehat{fg}_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-inx} f(x) g(x)\, dx$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} dx\, e^{-inx} \sum_k \hat{f}_k e^{+ikx} \sum_\ell \hat{g}_\ell e^{+i\ell x}$$

$$= \frac{1}{2\pi} \sum_{k,\ell} \hat{f}_k \hat{g}_\ell \underbrace{\int_{-\pi}^{\pi} dx\, e^{i(k+\ell-n)x}}_{\substack{=0 \text{ if } k+\ell \neq n \\ =2\pi \text{ if } k+\ell = n}} dx$$

$$= \sum_{k,\ell} \hat{f}_k \hat{g}_{n-k} .$$