Homework 5

A. Show that \(3x^2 - 2\sqrt{2}xy + 2y^2 = 1\) is the equation of an ellipse and find its semi-axes using linear algebra.

B. If \(A\) is negative definite and \(B = C^TAC\), is \(B\) necessarily negative definite?

C. Show that if \(A\) is positive definite then \(A^n\) is any integer power of \(A\)

D. If \(M\) is a real matrix is \(e^M\) necessarily positive definite?

E. Without using symbolic software find the eigenvalues and eigenvectors of \(\begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}\) \(v = 2\begin{bmatrix} 4 \\ 1 \end{bmatrix}\)

*(You can check your calculations using software).*