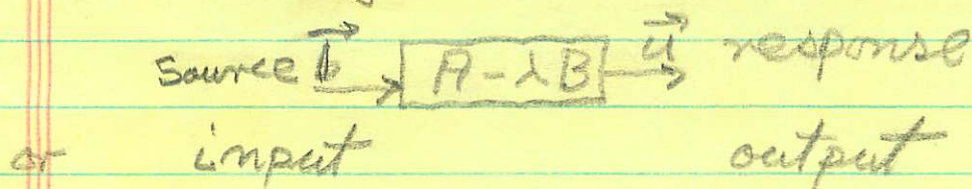


- 11.1 -

Review: The Linear Algebra  
behind the Green's function concept.

In dealing with a linear system  $A - \lambda B$  we are dealing with the

following causal relation



whose nature is found to be governed by the following equation

$$(A - \lambda B)\vec{u} = \vec{b}$$

The mathematical problem is this:

Given (a)  $A - \lambda B: U \rightarrow V$ ; (b) any  $b \in V$

Find  $\vec{u} \in U$  such that  $\boxed{(A - \lambda B)\vec{u} = \vec{b}}$  (\*)

Discussion: This linear algebra problem, it turns out, decomposes into answering two questions:

1. Can one find  $G$  such that  $(A - \lambda B)G = I$  on  $V$ .
2. Can one find  $H$  on  $U$  such that  $(A - \lambda B)^*H = I$ .



- 1.) Yes, one can find a right inverse of  $A - \lambda B$  if it has the property of being onto,  
i.e. for any  $b$ ,  $\exists$  a  $\vec{u}$  s.t.

$$(A - \lambda B)\vec{u} = b \quad (\text{existence})$$

i.e. the columns of  $A - \lambda B$  span  $V$

- 2.) Yes, if one can find  $H^*$ ,

$$\boxed{H^*(A - \lambda B) = I} \quad \text{on } U \quad (**)$$

i.e. can find a left inverse of  $A - \lambda B$ ,  
i.e.  $A - \lambda B$  has the property that

$$(A - \lambda B)u_1 = (A - \lambda B)u_2 \Rightarrow u_1 = u_2$$

$$(A - \lambda B)u = 0 \Rightarrow u = 0$$

i.e.  $A - \lambda B$  is 1-1, i.e.

i.e. the columns of  $A - \lambda B$  are independent,

$$3.) \quad 1.) \Rightarrow \underbrace{H^*(A - \lambda B)}_I G = H^* I$$

$$2.) \Rightarrow \left. \begin{array}{l} I G = H^* \\ ** \\ * \end{array} \right\} \Rightarrow$$

$$\boxed{\vec{u} = G \vec{b}}$$

SOL'N TO THE  
Problem.