Gaussian Integral (see also p91)

2/7/02

Evaluate $\int_{-\infty}^{\infty} e^{-ax^2+ikx}\,dx$ sol'n to # 4 of HWset 5

by completing the square

solution:

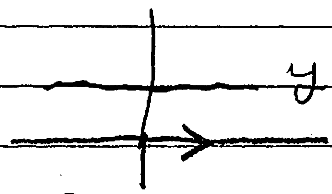
$$x^2-\frac{i}{a}kx = x^2-\frac{i}{a}kx+\left(\frac{-ik}{2a}\right)^2+\frac{k^2}{4a^2}$$

$$=\left(x-\frac{ik}{2a}\right)^2+\frac{k^2}{4a^2}$$

Thus

$$\int_{-\infty}^{\infty} e^{-a\left(x-\frac{ik}{2a}\right)^2} e^{-\frac{k^2}{4a}}\,dx \qquad \text{Let } y = x-\frac{ik}{2a}$$

$$=\int_{-\infty-\frac{ik}{2a}}^{\infty-\frac{ik}{2a}} e^{-ay^2}e^{-k^2/4a}\,dy.$$



By Cauchy-Goursat's theorem

$$=\int_{-\infty}^{\infty} e^{-ay^2}dy\; e^{-k^2/4a} = \sqrt{\frac{\pi}{a}}\,e^{-k^2/4a}$$