1 Sets

In mathematics, any collection or system of objects is called a set. The objects a set $A$ is made of are called elements of the set $A$. If $x$ is one of these objects we say that $x$ is an element of $A$ (or, that $x$ belongs to $A$), and denote $x \in A$.

If $y$ is not an element of $A$ we write $y \notin A$ (read: $y$ does not belong to $A$).

Examples of sets:
- the collection of chairs in room CL 137,
- the words on this page,
- an open interval $(a, b)$ (which is the collection of all real numbers $x$ satisfying $a < x < b$),
- the set consisting of the numbers 2, 3, 4, 5 and 6: \{2, 3, 4, 5, 6\} (when enumerating the elements of a set, you do so between \{ and \}),
- the integer numbers: \ldots, −3, −2, −1, 0, 1, 2, 3, . . .

The empty set is the set with no elements; it is denoted by $\emptyset$.

1.1 Subsets

Given two sets $A, B$ we say that $A$ is included in $B$ (or that $A$ is a subset of $B$) and denote $A \subset B$, if any element of $A$ is also an element of $B$:

$$A \subset B \quad \text{if and only if } \quad x \in A \implies x \in B$$

If $A \subset B$ we can also say that $B$ includes $A$, and denote $B \supset A$.

Examples:
1) The set of natural numbers is a subset of the real numbers.
2) $(1, 3) \subset [1, 3) \subset [1, 4)$
3) The empty set is included in any set: $\emptyset \subset A$.
4) $A \subset A$.

The notation $A \not\subset B$ means that the set $A$ is not included in $B$.

Example: \{-1, 0, 2, 3, 4\} \not\subset [0, +\infty)$. Why?
1.2 Operations with sets

**Union** \( A \cup B \) is the set which collects all elements of \( A \) and \( B \):

\[
x \in A \cup B \text{ if and only if } x \in A \text{ or } x \in B
\]

Examples:
1) \( \{0, 1, 2, 3, 4\} \cup \{3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\} \).
2) The domain of the function \( f(x) = 1/x \) is the set \( (-\infty, 0) \cup (0, +\infty) \).
3) \( \emptyset \cup A = A \)

Note: \( A \cup B = B \cup A \).

**Intersection** \( A \cap B \) is the set which collects all elements common to \( A \) and \( B \):

\[
x \in A \cap B \text{ if and only if } x \in A \text{ and } x \in B
\]

Examples:
1) \( \{0, 1, 2, 3, 4\} \cap \{3, 4, 5, 6\} = \{3, 4\} \)
2) \( [0, 2] \cap [1, 2] = [1, 2] \)
3) \( \emptyset \cap A = \emptyset \)

Note: \( A \cap B = B \cap A \).

**Difference** \( A \setminus B \) is the set which collects all elements of \( A \) which do not belong to \( B \):

\[
x \in A \setminus B \text{ if and only if } x \in A \text{ and } x \notin B
\]

Examples:
1) \( \{0, 1, 2, 3, 4\} \setminus \{3, 4, 5, 6\} = \{0, 1, 2\} \).
2) \( [0, 10] \setminus [1, 2] = [0, 1) \cup (2, 10] \)
3) \( A \setminus A = \emptyset \), \( A \setminus \emptyset = A \).
1.3 Exercises

1.1 Write, using a set notation, the domain of the following functions:

\[ f(x) = \sqrt{1 - x^2} \quad , \quad g(x) = \left( x^3 - 1 \right)^{1/3} \quad , \quad h(x) = \sqrt{x - 2} + \sqrt{4 - x} \]

1.2 Suppose the domain the function \( F(x) \) is the set \( A \), and the domain of the function \( G(x) \) is the set \( B \). Then the function \( F(x) + G(x) \) is certainly defined for numbers in which set?

1.3 Suppose the sets \( A \) and \( B \) are such that \( A \subset B \). Find the following sets: \( A \cap B, A \cup B, A \setminus B, B \setminus A \). Explain! (Draw a picture and explain in words.)

1.4 Suppose \( A \subset B \) and \( B \subset C \). Find \( A \cap C \). Explain! (Use both pictures and words.)

1.5 (A gentle work-out for the logical thinking muscle.)

Suppose \( Y \) is the set of yellow flowers in Mary’s garden, and \( R \) is the set of roses in Mary’s garden. What can you say about the flowers in Mary’s garden

1) if \( R \subset Y \) ? 
2) if \( R \not\subset Y \) ? 
3) if \( Y \subset R \) ? 
4) if \( Y \not\subset R \) ?
5) if \( R = Y \) ?

1.6 Denote by \( E \) the set of all even integers, and by \( T \) the set of all integers which are a multiple of 3 (so they are divisible by 3). Which of the following statements are true, and which are false? Explain!

1) \( 100 \in E \) 
2) \( 100 \in T \) 
3) \( 100 \in E \cup T \) 
4) \( 100 \in E \cap T \)
5) \( 100 \in E \setminus T \) 
6) \( 100 \in T \setminus E \)