1 Sets

In mathematics, any collection or system of objects is called a **set**.

The objects a set A is made of are called **elements of the set** A. If x is one of these objects we say that x is an element of A (or, that x *belongs to* A), and denote $x \in A$.

If y *is not* an element of A we write $y \notin A$ (read: y does not belong to A).

Examples of sets:
• the collection of chairs in room CL 137,
• the words on this page,
• an open interval (a, b) (which is the collection of all real numbers x satisfying $a < x < b$),
• the set consisting of the numbers 2, 3, 4, 5 and 6: $\{2, 3, 4, 5, 6\}$ (when enumerating the elements of a set, you do so between { and }),
• the integer numbers: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

The empty set is the set with no elements; it is denoted by $\emptyset$.

1.1 Subsets

Given two sets A, B we say that A **is included in** B (or that A is **a subset** of B) and denote $A \subset B$, if any element of A is also an element of B:

$$A \subset B \quad \text{if and only if} \qquad x \in A \;\; \text{implies} \;\; x \in B$$

If $A \subset B$ we can also say that B **includes** A, and denote $B \supset A$.

Examples:
1) The set of natural numbers is a subset of the real numbers.
2) $(1, 3) \subset [1, 3) \subset [1, 4)$
3) The empty set is included in any set: $\emptyset \subset A$.
4) $A \subset A$.

The notation $A \not\subset B$ means that the set A is *not included* in B.
Example: $\{-1, 0, 2, 3, 4\} \not\subset [0, +\infty)$. Why?

1.2 Operations with sets

Union $A \cup B$ is the set which collects all elements of A and B:

$$x \in A \cup B \text{ if and only if } x \in A \text{ or } x \in B$$

Examples:
1) $\{0, 1, 2, 3, 4\} \cup \{3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$.
2) The domain of the function $f(x) = 1/x$ is the set $(-\infty, 0) \cup (0, +\infty)$.
3) $\emptyset \cup A = A$
Note: $A \cup B = B \cup A$.

Intersection $A \cap B$ is the set which collects all elements common to A and B:

$$x \in A \cap B \text{ if and only if } x \in A \text{ and } x \in B$$

Examples:
1) $\{0, 1, 2, 3, 4\} \cap \{3, 4, 5, 6\} = \{3, 4\}$
2) $[0, 2] \cap [1, 2) = [1, 2)$
3) $\emptyset \cap A = \emptyset$
Note: $A \cap B = B \cap A$.

Difference $A \setminus B$ is the set which collects all elements of A which do not belong to B:

$$x \in A \setminus B \text{ if and only if } x \in A \text{ and } x \notin B$$

Examples:
1) $\{0, 1, 2, 3, 4\} \setminus \{3, 4, 5, 6\} = \{0, 1, 2\}$.
2) $[0, 10] \setminus [1, 2] = [0, 1) \cup (2, 10]$
3) $A \setminus A = \emptyset, \quad A \setminus \emptyset = A$.

1.3 Exercises

1.1 Write, using a set notation, the domain of the following functions:

$$f(x) = \sqrt{1-x^2} \quad , \quad g(x) = \left(x^3 - 1\right)^{1/3} \quad , \quad h(x) = \sqrt{x-2} + \sqrt{4-x}$$

1.2 Suppose the domain the function $F(x)$ is the set A, and the domain of the function $G(x)$ is the set B. Then the function $F(x) + G(x)$ is certainly defined for numbers in which set?

1.3 Suppose the sets A and B are such that $A \subset B$. Find the following sets: $A \cap B$, $A \cup B$, $A \setminus B$, $B \setminus A$.
Explain! (Draw a picture *and* explain in words.)

1.4 Suppose $A \subset B$ and $B \subset C$. Find $A \cap C$.
Explain! (Use both pictures and words.)

1.5 *(A gentle work-out for the logical thinking muscle.)*
Suppose Y is the set of yellow flowers in Mary's garden, and R is the set of roses in Mary's garden. What can you say about the flowers in Mary's garden
1) if $R \subset Y$? 2) if $R \not\subset Y$? 3) if $Y \subset R$?
4) if $Y \not\subset R$? 5) if $R = Y$?

1.6 Denote by E the set of all even integers, and by T the set of all integers which are a multiple of 3 (so they are divisible by 3). Which of the following statements are true, and which are false? *Explain!*

1) $100 \in E$ 2) $100 \in T$ 3) $100 \in E \cup T$ 4) $100 \in E \cap T$

5) $100 \in E \setminus T$ 6) $100 \in T \setminus E$