Some useful formulas

Some trigonometric formulas

\[ \sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \]

Some integration formulas

\[ \int \sqrt{x^2 + 1} \, dx = \frac{1}{2} x \sqrt{1 + x^2} + \frac{1}{2} \ln \left( x + \sqrt{1 + x^2} \right) + C \]

Conics

**Ellipse/Hyperbola** \( x^2/a^2 \pm y^2/b^2 = 1 \)

*In the case \( a > b \):* denote \( c = \sqrt{a^2 - b^2} \) for ellipse and \( c = \sqrt{a^2 + b^2} \) for hyperbola
and, for \( a > b \): Eccentricity: \( e = c/a \) Foci: \((\pm c, 0)\)

*In the case \( a < b \):* the roles of \( x \) and \( y \) are exchanged in the formulas above.

**Parabola:** \( x^2 = 4py \) Focus \((0, p)\).

Polar coordinates

Element of length: \( ds = \sqrt{dr^2 + r^2 d\theta^2} \)

Area bounded by \( r = r(\theta), \theta = \theta_0, \theta = \theta_1 \) is \( A = \int_{\theta_0}^{\theta_1} \frac{1}{2} r^2 d\theta \)

Angle \( \psi \): \( \tan \psi = \frac{r}{dr/d\theta} \)

Rotation of the rectangular coordinate axes \( Oxy \) by angle \( \theta \) to new coordinate axes \( Ox'y' \):

\( x = x' \cos \theta - y' \sin \theta, \quad y = x' \sin \theta + y' \cos \theta \)