A generalization of classical summation methods

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Among the many applications of classical summation formulas is to make sense of divergent representations (e.g. Borel summation), to transform unwieldy sums into easier to handle ones (Poisson summation) or into integrals expressions generally easier to analyze (Borel, Euler-Maclaurin and Abel-Plana summation). Another application is to provide constructive analytic continuation, when this is possible. This is important since Taylor series are local objects, poorly suited for global analysis. Questions such as: does $f(z) = \displaystyle\sum_{k=1}^{\infty} \frac{z^n}{n + \ln\ln(n)}$ possess analytic continuation on the universal covering of $\mathbb{C} \setminus S$ where S is some discrete set, and if so, what is the behavior of f as $z \to \infty$? what is S and what is the nature of the singularities of f? or, what is the behavior of the "approximate exponential" $1+\displaystyle\sum_{k=1}^{\infty} \frac{z^k}{\sqrt{2\pi k}(k/e)^k}$ as $z \to -\infty$? are hard to answer without changing somehow the representation of the functions.

I will present a summation method that apply to fairly general series, in particular to series whose coefficients satisfy generic linear or nonlinear difference equations, and generalizes the classical ones mentioned above.

The method also uncovers interesting dualities between the analytic properties of a function of a complex, given by a series, variable and those of its coefficients c_n as a function of n.