Math 5101 HOMEWORK 10

Recall the following facts on the complex roots of 1:

The n complex solutions of the equation $z^n = 1$, called *the* n^{th} *roots of unity*, are $z_k = \exp[2\pi ik/n],\ k = 0, 1, \ldots, n-1$.

z_1 is called the *primitive* n^{th} *root of unity*. We have $z_k = z_1^k$.

Recall the formula $z^n - 1 = (z-1)(z^{n-1} + z^{n-2} + \ldots + z + 1)$. This shows that $z_1, z_2, \ldots, z_{n-1}$ satisfy $z^{n-1} + z^{n-2} + \ldots + z + 1 = 0$.

1. Let $w = e^{2\pi i/n}$ be the primitive n^{th} root of unity.

Consider the matrix

$$U = \frac{1}{\sqrt{n}}[w^{jk}]_{j,k=0,1,\ldots,n-1}$$

which is, in fact, the Fourier Matrix:

$$U = \frac{1}{\sqrt{n}}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{n-1} \\ 1 & w^2 & w^4 & \cdots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \ldots & w^{(n-1)^2} \end{bmatrix}$$

Show that U is unitary.

2. Find the matrix P which takes any vector in $\mathbb{R}^4$ to its orthogonal projection on the (2-d) plane spanned by the vectors $(1, 4, 4, 1)$ and $(2, 9, 8, 2)$.

3. Find the QR factorization of the matrix $\begin{bmatrix} \cos t & \sin t \\ \sin t & 0 \end{bmatrix}$. Show all the steps in your calculations, made without the use of a symbolic calculation software.

4. If Q is an orthogonal matrix, is the same true for Q^5?