Math 5101 **Homework 12**

Name(s):

1. If every entry in an orthogonal matrix is either $\frac{1}{4}$ or $-\frac{1}{4}$, how big is the matrix?

2. Write the rank one matrix $M = \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$ as $\mathbf{x}\mathbf{y}^T$ and write M^T in the same form.

3. Consider $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Find the pseudoinverse M^+ and the optimal solution of the system $M\mathbf{x} = \mathbf{b}$.

4. a) If MM^* is invertible show that $M^+ = M^*(MM^*)^{-1}$ in the following steps:
(i) the system $M\overline{\mathbf{x}} = \mathbf{b}$ has solutions for all $\mathbf{b}$,
(ii) denoting temporarily $M_+ = M^*(MM^*)^{-1}$ show that if $\overline{\mathbf{x}} = M_+\mathbf{b}$ then $M\overline{\mathbf{x}} = \mathbf{b}$,
(iii) $\overline{\mathbf{x}}$ defined above belongs to $\mathcal{N}(M)^\perp$.

b) With $M = [1 \;\; 1]$ find the optimal solution to $x + y = 3$.

5. Find the pseudoinverse of $M = [3 \;\; 0]$.