5101 (AU 2017) **Homework 3** Name(s):

1. Show that if the rows of a determinant of order n are linearly dependent, then also its columns are linearly dependent.

2. Let $A_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$, $A_3 = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 2 \end{bmatrix}$, $Y = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix}$, $Z = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 4 \end{bmatrix}$.

a1) True or False? $Y \in Sp(A_1, A_2, A_3)$.
a2) True or False? $Z \in Sp(A_1, A_2, A_3)$.
b) Find a basis for $Sp(A_1, A_2, A_3)$.
Justify all your answers!

3. Let $\mathcal{M}_{2,2}(\mathbb{R})$ be the linear space of all 2×2 matrices with the real entries. Prove that $\mathcal{M}_{2,2}(\mathbb{R})$ has dimension 4 and find a basis.

4. a) Consider the multiplication transformation $R_t : \mathbb{C} \to \mathbb{C}$ defined by $R_t z = e^{it} z$. Show that this is a linear transformation. Use the polar form of complex numbers to show that this is a rotation of angle t of the complex plane.

b) Every complex number has the form $z = x_1 + ix_2$ with $x_{1,2} \in \mathbb{R}$ uniquely determined, so we can think of $\mathbb{C}$ as $\mathbb{R}^2$ by identifying every $z \in \mathbb{C}$ with $(x_1, x_2) \in \mathbb{R}^2$. Write formulas for the rotation R_t as a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$. Find its matrix in the standard basis

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

5. Consider a linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $T\mathbf{e}_1 = \mathbf{e}_3$ and $T\mathbf{e}_2 = 3\mathbf{e}_1$. Find the matrix of this transformation in standard bases and a formula for $T\mathbf{x}$.