5101 (AU 2017) **Homework 7** Name(s):

Always justify your answers!

1. Let M be an invertible matrix. Assume M has n independent eigenvectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$, corresponding to the eigenvalues $\lambda_1, \ldots, \lambda_n$.

a) If $\mathbf{v}$ is an eigenvector of M, show that $\mathbf{v}$ is also an eigenvector of M^2 and, if the eigenvalue is nonzero, also of M^{-2}. What are the corresponding eigenvalues?

b) Find the eigenvectors and eigenvalues of M^k for any integer k.

c) Find the eigenvectors and eigenvalues of $M + cI$ (where c is a scalar and I is the identity matrix).

d) Find the eigenvectors and eigenvalues of $M^2 - 10M + 25I$.

2. For each of the matrices A, B below: find the eigenvalues and eigenvectors.
If the matrix is diagonalizable explain why this is the case, find its diagonal form and a transition matrix. Otherwise, find a Jordan normal form and a transition matrix.

$$A = \begin{bmatrix} 5 & 4 & 2 & -4 \\ 0 & 1 & 0 & 0 \\ -2 & -2 & 0 & 2 \\ 2 & 2 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 5 & 2 & -4 \\ 0 & 1 & 0 & 0 \\ -2 & -2 & 0 & 2 \\ 2 & 3 & 1 & -1 \end{bmatrix}$$

3. Solve problems number 5.2.n with $n \in \{1, 3, 5, 6, 7, 8, 9, 10,\}$ from the handout of Wed. Oct. 4.