Some review questions

1. *True or false? If true, justify, if false, give a counterexample.*
(a) A unitary matrix is always diagonalizable.
(b) A square matrix with all eigenvalues real is always diagonalizable.
(c) A square matrix which is not symmetric is not diagonalizable.
(d) A square matrix which is diagonalizable by a unitary transformation must be normal.

2. *True or false? If true, justify, if false, give a counterexample.*
(a) Any matrix is similar to a diagonal matrix.
(b) Any square matrix is unitarily similar to a triangular matrix.
(c) Any non-singular square matrix can be written as a product of a unitary and a positive definite self adjoint matrix.
(d) Any symmetric positive definite matrix has a square root.
(e) Self-adjoint matrices are invertible.

3. *If N is a normal matrix, then which of the following are correct?*
(a) $N = N^*$
(b) $N = (N^*)^{-1}$
(c) N is diagonalizable.
(d) N has only real eigenvalues.
(e) All the powers of N are normal.
(f) N^*N is selfadjoint.
(g) None of the above is correct.

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4. *If A is a (real) symmetric matrix, then which of the following are correct?*
(a) A is normal.
(b) If A is orthogonal, then its eigenvalues can only be 1 or -1.
(c) $1-3i$ cannot be an eigenvalue of A.
(d) AA^T is always positive semidefinite.
(e) None of the above is correct.

7. *True or False? Give the reason if true or give a counterexample if false.*
(a) For every square matrix M there is a unique solution to $\dfrac{du}{dt} = Mu$ with the initial condition $u_0 = (1, \dots, 1)^T$.
(b) Every invertible matrix can be diagonalized.
(c) Every diagonalizable matrix can be inverted.
(d) The eigenvalues of M^* are the complex conjugates of the eigenvalues of M.
(e) If the eigenvectors $\mathbf{v}$ and $\mathbf{y}$ correspond to distinct eigenvalues, then $\langle \mathbf{v}, \mathbf{y} \rangle = 0$.