REVIEW

Review the theory:

You need to know (to state and use) all the theorems we proved in class.
You need to know how to prove the following theorems:
3.2.3, 3.2.4, 3.2.7, 3.2.11,
3.3.2,
3.5.5, 3.5.8,
3.6.3, 3.6.4, 3.6.5
4.1.8,
4.2.2, 4.2.4, 4.2.7, 4.2.9,
4.3.2,
5.2.1, 5.2.6,
5.3.2, 5.3.4, 5.3.10
5.4.3, 5.4.5, 5.4.8, 5.4.10
5.6.1
Show that the $n$the root exists and it is continuous.

Review Problems

A. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and periodic. Does it imply that $f(x)$ is bounded?

B. Prove Brouwer fixed point theorem: Any continuous function $f : [a, b] \to [a, b]$ has a fixed point, that is, an x for which $f(x) = x$.

C. Consider two continuous functions $f : [0, 1] \to [0, 1]$ and $g : [0, 1] \to [0, 1]$ such that$f(0) \geqslant g(0)$ and $f(1) \leqslant g(1)$. Prove that there exists $x \in [0, 1]$ such that $f(x) = g(x)$.

D. Let $f : [0, 2] \to \mathbb{R}$ be continuous, $f(0) = f(2)$. Prove that the graph of f has a chord of length 1.

E. Let f be continuous on an interval I, $f : I \to \mathbb{R}$. Which of the following sets *cannot* equal $f(I)$: $A = [0, 1]$, $B = \{0, 1\}$, $C = \{0\}$, $D = \{1\}$, $E = [0, \infty)$, $F = \mathbb{R}$?

F. Prove that the polynomial $x^3 - 3x^2 - x + 2$ has 3 real roots.

G. Prove that if $f : [a, b] \to \mathbb{R}$ is continuous, and $x_1, x_2, \ldots, x_n \in [a, b]$ then there is $c \in [a, b]$ for which $f(c) = \frac{f(x_1)+f(x_2)+\ldots+f(x_n)}{n}$.

And More problems:

1. Prove that if $f, g : \mathbb{R} \to \mathbb{R}$ are continuous and for some $a \in \mathbb{R}$ we have $f(a) < g(a)$ then $f(x) < g(x)$, for all x in some neighborhood of a.

---

2. Prove that if $f, g : A \to \mathbb{R}$ are continuous at $a \in A$ then $\max\{f, g\}$ is cont. at a.

---

3. Can we extend $\frac{\sqrt{x}-1}{x-1}$ to $x=1$ continuously?

---

4. At which points is the following function continuous

$$f(x) = \begin{cases} \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

---

5. At which points is the function continuous

$$g(x) = \begin{cases} x \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

---

6. At which points is the function continuous

$$h(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

---

7. Find $\lim_{x \to 0} \frac{\sin 3x}{\sin 5x}$

8 $\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)=?$

---

9. $\lim_{x\to\infty}\dfrac{\sin x^{10}}{x}=?$

---

10. $\lim_{x\to\infty}\dfrac{x+\cos x}{\sqrt{x^2+x+1}}$

Also Problems Sec 5.1 # 8, 10

Sec 5.2 #10, 14

Sec 5.3 # 1, 4, 5 (just locate the roots within nearest integers)
11, 14

Sec 5.4 #1, 4, 6, 8, 9, 10, 15