

Hints

1. is trivial
2. factor, simplify
3. factor the highest power of x at the numerator and at the denominator

4. what does \sin do?

5. Split the fraction as a sum

6. Use $a-b = \frac{a^3-b^3}{a^2+ab+b^2}$

7. Find the lateral limits

8.
$$\frac{x^{100} - x - x + 1}{x^{50} - x - x + 1} = \frac{x(x^{99} - 1) - (x-1)}{x(x^{49} - 1) - (x-1)}$$
 and factor