More review questions

State the following:

1. Let $A \subseteq \mathbb{R}$, $A \neq \emptyset$. State the definition of lower bound of A, upper bound of A, $\inf A$, $\sup A$.

2. Let (x_n) be a sequence of real numbers. State the definition of x_n is convergent. is Cauchy

2'. State Bolzano-Weierstrass theorem.

3. Let $f: A \to \mathbb{R}$ and c be a cluster point of A. State the definition of $\lim_{x \to c} f = L$

4. State the definition of "c is a cluster point of the set $A \subseteq \mathbb{R}$".

5. State the Nested intervals Property

6. State Bolzano's Intermediate Value Theorem

7. State the Preservation of intervals theorem

8. State the Continuous inverse Theorem.

9. State the Continuous Extension Theorem.

10. State the Uniform Continuity theorem.

See next page

True or False? — (Yes) — needs a proof
(NO) needs a counterexample

10. Every sequence of real numbers has a bounded subsequence.

11. Every sequence has a convergent subsequence.

12. Every function defined on a closed and bounded interval, $f : [a,b] \to \mathbb{R}$ is continuous

13. If $f, g : A \to \mathbb{R}$ are uniformly continuous on A then also
a) $f+g$ is unif. cont
b) $f-g$ is unif cont
c) fg is unif cont

14. If $f : \mathbb{R} \to \mathbb{R}$ is uniformly continuous, then f is bounded.

15 If $f : [a,b] \to \mathbb{R}$ is unif cont then f is bounded.

16. If $f : \mathbb{R} \to \mathbb{R}$ is Lipschitz then f is continuous on $\mathbb{R}$

17. If $f : \mathbb{R} \to \mathbb{R}$ is continuous, then f is Lipschitz.

18. If $f : \mathbb{R} \to \mathbb{R}$ is Lipschitz then the sequence defined recursively by $x_1 = 1$, $x_{n+1} = f(x_n)$ is contractive.

19. A Cauchy sequence of real numbers is bounded.

20. If $f : \mathbb{R} \to \mathbb{R}$ has the property that $\lim_{x \to c} f(x)$ exists, $= L \in \mathbb{R}$ then f is bounded in a neighborhood of c.

21. If $f : A \to \mathbb{R}$ is continuous and $f(x_0) > \alpha$ then there exists a neighborhood of x_0 such that $f(x) > \alpha$ for all x in this neighborhood.

22. There exists a decreasing function on $\mathbb{R}$ which is discontinuous at all the irrational points.

see next page

101. Let $f:(0,\infty)\to\mathbb{R}$, $f(x)=\frac{1}{x}$.
Give a careful, ε-δ proof that f is continuous at $x=1$.

102. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=\begin{cases} x^2, & x\geq 0 \\ x, & x<0 \end{cases}$
At which points is f continuous? Give detailed explanations

103. Let $g:(-1,1)\to\mathbb{R}$ be defined by $g(x)=\begin{cases} 2x, & \text{for } 0\leq x<1 \\ x^2, & \text{for } -1<x<0 \end{cases}$
Is g uniformly continuous on $(-1,1)$?

104. Prove, using ε-δ arguments, that if $f,g:A\to\mathbb{R}$, and c is a cluster point of A, if $\lim\limits_{x\to c} f=L$ and $\lim\limits_{x\to c} g=M$ then $\lim\limits_{x\to c} fg=LM$.