1. Solve the initial value problem

$$y'' + 4y = 0; \quad y(0) = 0,\ y'(0) = 1$$

2. A mechanical oscillator with $m = 1$, $\gamma = 2$, $k = 1$ starts at $t = 0$ in the equilibrium position, $x = 0$, with velocity 1. What is the maximal displacement x? What is the behavior of the solution as $t \to \infty$?.

3. (a) In an RLC circuit, $L = C = 1$. For which range of R is the oscillator **overdamped?**

   (b) Choose now $R = 0$ and assume an external voltage $V(t) = \sin(5t/4)$ is applied to the circuit in (a) and that the initial current is $-4/9$. What is the frequency of the beats?

4. Find the general solution of the equation

$$\varphi'' + 2\varphi' + \varphi = e^x + xe^{-x}$$

5. Consider the equation

$$(2x-1)f''(x) - \left(1 + 4\,x^2\right) f'(x) + \left(2 + 4\,x^2 - 2\,x\right) f(x) = 0 \quad (*)$$

   (a) What is the guaranteed interval of existence of the solution of (*) with $f(1) = 0, f'(1) = 0$?

   (b) Check that a particular solution of (*) is e^x. Find a second solution, linearly independent from e^x.

   (c) What is the **actual** interval of existence of of the solution of (*) with $f(1) = 0, f'(1) = 0$? Compare with the answer to (a).

   (d) Find the general solution of

$$f''(x) - \frac{(1 + 4\,x^2)}{2\,x - 1} f'(x) + \frac{(2 + 4\,x^2 - 2\,x)}{2\,x - 1} f(x) = x(2x-1)$$

6. Find two linearly independent solutions of the equation below, as power series centered at zero.

$$y'' + x^2 y = 0$$

What is the radius of convergence of the series that you obtained?

Bonus: Assume that $y(x) \to L$ as $x \to +\infty$. Show that $L = 0$.

7. Consider the differential equation $(x^2+1)y'' + y(x) = 0$ with the initial condition $y(0) = 1, y'(0) = 0$. What is the guaranteed interval of existence of this solution?

   (b) Find y as a power series. What is the radius of convergence of the series? Compare with (a).