1. Solve the initial value problem
\[ y'' + 4y = 0; \quad y(0) = 0, \quad y'(0) = 1 \]

2. A mechanical oscillator with \( m = 1, \gamma = 2, k = 1 \) starts at \( t = 0 \) in the equilibrium position, \( x = 0 \), with velocity 1. What is the maximal displacement \( x \)? What is the behavior of the solution as \( t \to \infty \)?

3. (a) In an RLC circuit, \( L = C = 1 \). For which range of \( R \) is the oscillator **overdamped**?

   (b) Choose now \( R = 0 \) and assume an external voltage \( V(t) = \sin(5t/4) \) is applied to the circuit in (a) and that the initial current is \(-4/9\). What is the frequency of the beats?

4. Find the general solution of the equation
\[ \varphi'' + 2\varphi' + \varphi = e^x + xe^{-x} \]

5. Consider the equation
\[ (2x - 1)f''(x) - \left( 1 + 4x^2 \right) f'(x) + \left( 2 + 4x^2 - 2x \right) f(x) = 0 \quad (\text{\#}) \]

   (a) What is the guaranteed interval of existence of the solution of (\#) with \( f(1) = 0, f'(1) = 0 \)?

   (b) Check that a particular solution of (\#) is \( e^x \). Find a second solution, linearly independent from \( e^x \).

   (c) What is the **actual** interval of existence of the solution of (\#) with \( f(1) = 0, f'(1) = 0 \)? Compare with the answer to (a).

   (d) Find the general solution of
\[ f''(x) - \frac{(1 + 4x^2)}{2x - 1} f'(x) + \frac{(2 + 4x^2 - 2x)}{2x - 1} f(x) = x(2x - 1) \]

6. Find two linearly independent solutions of the equation below, as power series centered at zero.
\[ y'' + x^2y = 0 \]
What is the radius of convergence of the series that you obtained?

Bonus: Assume that \( y(x) \to L \) as \( x \to +\infty \). Show that \( L = 0 \).

7. Consider the differential equation \((x^2 + 1)y'' + y(x) = 0\) with the initial condition \( y(0) = 1, y'(0) = 0 \). What is the guaranteed interval of existence of this solution?

(b) Find \( y \) as a power series. What is the radius of convergence of the series? Compare with (a).