1. $y(x) = \frac{1}{2}\sin 2x$

2. e^{-1}; The solution goes to zero.

3. (a) $R \in (2, \infty)$; (b) the solution is $-\frac{32}{9}\sin(t/8)\cos(9t/8)$ and thus the frequency is $\frac{1}{16\pi}$.

4. $\frac{1}{4}e^x + \frac{x^3}{6}e^{-x} + C_1 e^{-x} + C_2 x e^{-x}$

5. (a) $(1/2, \infty)$; (b) a second solution is e^{x^2}. (c) The actual interval of existence is $(-\infty, \infty)$, since e^x and e^{x^2} are smooth on $(-\infty, \infty)$; (d): $x + \frac{1}{2} + C_1 e^x + C_2 e^{x^2}$

6. $c_0 y_0 + c_1 y_1$ where $y_0 = \sum_{k=0}^{\infty} b_{4k} x^{4k}$ and $y_1 = \sum_{k=0}^{\infty} b_{4k+1} x^{4k+1}$, c_0, c_1 are arbitrary. The recurrence relation is $b_{k+2} = -\frac{b_{k-2}}{(k+1)(k+2)}$, $b_0 = 1, b_1 = 1, b_2 = 0, b_3 = 0$, thus $b_{4k+2} = b_{4k+3} = 0$. The radius of convergence is infinite, by the ratio test.

7. Written in the form required by Theorem 3.2.1, $y'' + \frac{1}{1+x^2}y = 0$, we see that the coefficients are continuous everywhere and thus by Theorem 3.2.1 the solutions exist on $(-\infty, \infty)$.

8. $y = c_0 y_0 + c_1 y_1$ where $y_0 = \sum_{k=0}^{\infty} b_{2k} x^{2k}$, $y_0 = \sum_{k=0}^{\infty} b_{2k+1} x^{2k+1}$ and c_0, c_1 are arbitrary. The recurrence relation is $b_{k+2} = -b_k \frac{k^2 - k + 1}{(k+1)(k+2)}$, $b_0 = 1$, $b_1 = 1$. The radius of convergence is one, and it is calculated for instance by the ratio test.