

MORE $\varepsilon - \delta$ PROBLEMS

1. Look back at the way we determined the limit $\lim_{x \rightarrow 0} \frac{\sin x}{x}$. Find the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

and write a careful $\varepsilon - \delta$ proof of your result.

2. Denote $f(x) = \text{frac}(x)$ the fractional part of x (that is the number between zero and one obtained by subtracting the integer part of x from x). Does $\lim_{x \rightarrow 1} f(x)$ exist? Write an $\varepsilon - \delta$ proof of your conclusion.