BONUS PROBLEM, 10 POINTS

Assume f and g are two functions defined on $\mathbb{R}$ with the properties

1. $\lim\limits_{x \to 0} f(x)/x = 1$

2. If $|x| < 1/2$ then $g(x) > 0$.

3. $f^2 + g^2 = 1$.

(An example is of course $f = \sin, g = \cos$.)

Show that $\lim\limits_{x \to 0} \dfrac{1 - g(x)}{x^2} = 1/2$.