Name :_____

Practice Midterm 1, 4181H

You must show all work on all problems to receive full credit.

Do not use calculators, textbooks, or other notes.

Justify your answers. You can use any theorem you learned as well as facts proved in homework problems. Many problems admit simple solutions, so if the calculations get very involved, there is probably a better approach. You may be able to find a solution which does not use results from the book; such a solution is perfectly acceptable as long as it is rigorous.

Problem 5 is a bonus problem. It can increase your score by up to 20p; however, the total caps at 110p.

Signature:_____

Problem	Possible	Received
1.	20	
2.	30	
3.	30	
4.	20	
5 (bonus)	20	
Total	100–110	
Bonus	20	

1. (1) Show that for any $x \in \mathbb{R}$ there is a $y \in \mathbb{R}$ such that $y^3 = x$.

(2) Show that for all $x, y \in \mathbb{R}$ we have $x^3 > y^3$ if and only if $x > y$.

2. Let $a_1 = 1/2$ and, for every $n \in \mathbb{N}$, let

$$a_{n+1} = a_n(1 - a_n)$$

Show that $0 < a_n < 1$ for all $n \in \mathbb{N}$.

3. True or false? If false, provide a counterexample; if true, give a proof.

" Assume that both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} f(x)g(x)$ exist. Then $\lim_{x\to 0} g(x)$ exists as well."

4. Let $a > 0$. Given an $\varepsilon > 0$ *find* a $\delta > 0$ such that $0 < |x - a| < \delta$ implies $|a^3 - x^3| < \varepsilon$. You don't have to find "the best δ", just any δ that works.

5. (Bonus) Prove or disprove:

"The function f given by $f(x) = \dfrac{1}{x^2+1}$ is uniformly continuous on $\mathbb{R}$ (that is, for any ε there is a δ such that for *any* pair of real numbers (x, y), if $|x - y| < \delta$ then $|f(x) - f(y)| < \varepsilon$".