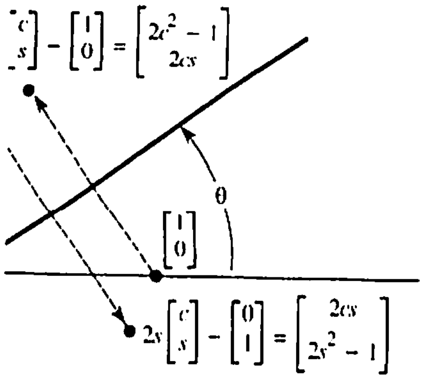
ıas no inverse, because the transformation has no inverse. Points like ıe perpendicular line are projected onto the origin; that line is the P. At the same time, points on the θ-line are projected to themselves! ds, projecting twice is the same as projecting once, and $P^2 = P$:

$$P^2 = \begin{bmatrix} c^2 & cs \\ cs & s^2 \end{bmatrix}^2 = \begin{bmatrix} c^2(c^2+s^2) & cs(c^2+s^2) \\ cs(c^2+s^2) & s^2(c^2+s^2) \end{bmatrix} = P.$$

$+ s^2 = \cos^2\theta + \sin^2\theta = 1$. ***A projection matrix equals its own square.*** ımetric.

Figure 2.10 shows the reflection of (1, 0) in the θ-line. The length on equals the length of the original, as it did after rotation—but those ɔns are very different. Here the θ-line stays where it is. The perpen- everses direction; all points go straight through the mirror. Linearity əst.



$$H = \begin{bmatrix} 2c^2 - 1 & 2cs \\ 2cs & 2s^2 - 1 \end{bmatrix}$$

lection through the θ-line: the geometry and the matrix.

H has the remarkable property $H^2 = I$. ***Two reflections bring back*** Thus a reflection is its own inverse, $H = H^{-1}$, which is clear from the : less clear from the matrix. One approach is through the relationship to projections: $H = 2P - I$. This means that $Hx + x = 2Px$—the ıe original equals twice the projection. It also confirms that

$$H^2 = (2P - I)^2 = 4P^2 - 4P + I = I,$$

ections satisfy $P^2 = P$.

:e transformations either leave lengths unchanged (rotations and re- reduce the length (projections). Other transformations can increase retching and shearing are in the exercises. Each example has a matrix it—which is the main point of this section. But there is also the question of choosing a basis, and we emphasize that *the matrix depends on the choice of basis.* For example:

(i) For projections, suppose the first basis vector is *on the θ-line* and the second basis vector is perpendicular. Then the projection matrix is back to $P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. This matrix is constructed as always: its first column comes from the first basis vector (which is projected to itself), and the second column comes from the basis vector which is projected onto zero.

(ii) For reflections, that same basis gives $H = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. The second basis vector is reflected onto its negative, to produce this second column. The matrix H is still $2P - I$, when the same basis is used for H and P.

(iii) For rotations, we could again choose unit vectors along the θ-line and its perpendicular. But the matrix would not be changed. Those lines are still rotated through θ, and $Q = \begin{bmatrix} c & -s \\ s & c \end{bmatrix}$ as before.

The whole question of choosing the best basis is absolutely central, and we come back to it in Chapter 5. The goal is to make the matrix diagonal, as achieved for P and H. To make Q diagonal requires complex vectors, since all real vectors are rotated.

We mention here the effect on the matrix of a *change of basis*, while the linear transformation stays the same. The matrix A (or Q or P or H) ***is altered to*** $S^{-1}AS$. Thus a single transformation is represented by different matrices (via different bases, accounted for by S). The theory of eigenvectors will lead to this formula $S^{-1}AS$, and to the best basis.

EXERCISES

2.6.1 What matrix has the effect of rotating every vector through 90° and then projecting the result onto the x-axis?

2.6.2 What matrix represents projection onto the x-axis followed by projection onto the y-axis?

2.6.3 Does the product of 5 reflections and 8 rotations of the x-y plane produce a rotation or a reflection?

2.6.4 The matrix $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ produces a ***stretching*** in the x-direction. Draw the circle $x^2 + y^2 = 1$ and sketch around it the points $(2x, y)$ that result from multiplication by A. What shape is that curve?

2.6.5 Every straight line remains straight after a linear transformation. If z is halfway between x and y, show that Az is halfway between Ax and Ay.

2.6.6 The matrix $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ yields a ***shearing*** transformation, which leaves the y-axis unchanged. Sketch its effect on the x-axis, by indicating what happens to (1, 0) and (2, 0) and (−1, 0)—and how the whole axis is transformed.

2.6.7 What 3 by 3 matrices represent the transformations that

i) project every vector onto the x–y plane?
ii) reflect every vector through the x–y plane?

iii) rotate the x–y plane through 90°, leaving the z-axis alone?
iv) rotate the x–y plane, then the x–z plane, then the y–z plane, all through 90°?
v) carry out the same three rotations, but through 180°?

2.6.8 On the space P_3 of cubic polynomials, what matrix represents d^2/dt^2? Construct the 4 by 4 matrix from the standard basis $1, t, t^2, t^3$. What is its nullspace, what is its column space, and what do they mean in terms of polynomials?

2.6.9 From the cubics P_3 to the fourth degree polynomials P_4, what matrix represents multiplication by $2 + 3t$? The columns of the 5 by 4 matrix A come from applying the transformation to each basis vector $x_1 = 1, x_2 = t, x_3 = t^2, x_4 = t^3$.

2.6.10 The solutions to the linear differential equation $d^2u/dt^2 = u$ form a vector space (since combinations of solutions are still solutions). Find two independent solutions, to give a basis for that space.

2.6.11 With initial values $u = x$ and $du/dt = y$ at $t = 0$, what combination of basis vectors in Ex. 2.6.10 solves the equation? This transformation from the initial values to the solution is linear; what is its 2 by 2 matrix (using $x = 1, y = 0$ and $x = 0, y = 1$ as basis for V, and your basis for W)?

2.6.12 Verify directly from $c^2 + s^2 = 1$ that the reflection matrices satisfy $H^2 = I$.

2.6.13 Suppose A is a linear transformation from the x–y plane to itself. Show that A^{-1} is also a linear transformation (if it exists). If A is represented by the matrix M, explain why A^{-1} is represented by M^{-1}.

2.6.14 The product $(AB)C$ of linear transformations starts with a vector x, produces a vector $u = Cx$, and then follows the shaded rule 2V in applying AB to u. It reaches $(AB)Cx$.

i) Is the result the same as separately applying C then B then A?
ii) Is the result the same as applying BC followed by A? If so, parentheses are unnecessary and the associative law $(AB)C = A(BC)$ holds for linear transformations. Combined with the product rule 2V, this is the best proof of the same law for matrices.

2.6.15 Prove that A^2 is a linear transformation if A is (say from $\mathbf{R}^3$ to $\mathbf{R}^3$).

2.6.16 The space of all 2 by 2 matrices has the four basis "vectors"

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Consider the linear transformation of *transposing* every 2 by 2 matrix, and find its matrix A with respect to this basis. Why is $A^2 = I$?

2.6.17 Find the 4 by 4 matrix that represents a cyclic permutation: each vector (x_1, x_2, x_3, x_4) is transformed to (x_2, x_3, x_4, x_1). What is the effect of A^2? Show that $A^3 = A^{-1}$.

2.6.18 Find the 4 by 3 matrix A that represents a *right shift*: each vector (x_1, x_2, x_3) is transformed to $(0, x_1, x_2, x_3)$. Find also the *left shift* matrix B from $\mathbf{R}^4$ back to $\mathbf{R}^3$, transforming (x_1, x_2, x_3, x_4) to (x_2, x_3, x_4). What are the products AB and BA?

2.6.19 In the vector space V of all cubic polynomials $P = a_0 + a_1x + a_2x^2 + a_3x^3$, let S be the subset of polynomials with $\int_0^1 p(x)\,dx = 0$. Verify that S is a subspace and find a basis.

2.6.20 A *nonlinear* transformation is invertible if there is existence and uniqueness; $f(x) = b$ has exactly one solution for every b. The example $f(x) = x^2$ is not invertible because $x^2 = b$ has two solutions for positive b and no solution for negative b. Which of the following transformations (from the real numbers $\mathbf{R}^1$ to the real numbers $\mathbf{R}^1$) are invertible? None are linear, not even (c).
(a) $f(x) = x^3$ (b) $f(x) = e^x$ (c) $f(x) = x + 11$ (d) $f(x) = \cos x$.

2.6.21 What is the axis of rotation, and the angle of rotation, of the transformation that takes (x_1, x_2, x_3) into (x_2, x_3, x_1)?

REVIEW EXERCISES: Chapter 2

2.1 Find a basis for the following subspaces of $\mathbf{R}^4$:
(a) The vectors for which $x_1 = 2x_4$
(b) The vectors for which $x_1 + x_2 + x_3 = 0$ and $x_3 + x_4 = 0$
(c) The subspace spanned by (1, 1, 1, 1), (1, 2, 3, 4), and (2, 3, 4, 5).

2.2 By giving a basis, describe a two-dimensional subspace of $\mathbf{R}^3$ that contains none of the coordinate vectors (1, 0, 0), (0, 1, 0), (0, 0, 1).

2.3 True or false, with counterexample if false:
(i) If the vectors $x_1, \ldots, x_m$ span a subspace S, then dim $S = m$.
(ii) The intersection of two subspaces of a vector space cannot be empty.
(iii) If $Ax = Ay$, then $x = y$.
(iv) The row space of A has a unique basis that can be computed by reducing A to echelon form.
(v) If a square matrix A has independent columns, so does A^2.

2.4 What is the echelon form U of

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix}?$$

What are the dimensions of its four fundamental subspaces?

2.5 Find the rank and the nullspace of

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

2.6 Find bases for the four fundamental subspaces associated with

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

2.7 What is the most general solution to $u + v + w = 1$, $u - w = 2$?

2.8 (a) Construct a matrix whose nullspace contains the vector $x = (1, 1, 2)$.
(b) Construct a matrix whose left nullspace contains $y = (1, 5)$.
(c) Construct a matrix whose column space is spanned by (1, 1, 2) and whose row space is spanned by (1, 5).
(d) If you are given any three vectors in $\mathbf{R}^6$ and any three vectors in $\mathbf{R}^5$, is there a 6 by 5 matrix whose column space is spanned by the first three and whose row space is spanned by the second three?

2.9 In the vector space of 2 by 2 matrices,
(a) is the set of rank-one matrices a subspace?
(b) what subspace is spanned by the permutation matrices?
(c) what subspace is spanned by the positive matrices (all $a_{ij} > 0$)?
(d) what subspace is spanned by the invertible matrices?

2.10 Invent a vector space that contains all linear transformations from $\mathbf{R}^n$ to $\mathbf{R}^n$. You ha to decide on a rule for addition. What is its dimension?

2.11 (a) Find the rank of A, and give a basis for its nullspace.

$$A = LU = \begin{bmatrix} 1 & & & \\ 2 & 1 & & \\ 2 & 1 & 1 & \\ 3 & 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) T F The first 3 rows of U are a basis for the row space of A
T F Columns 1, 3, 6 of U are a basis for the column space of A
T F The four rows of A are a basis for the row space of A
(c) Find as many linearly independent vectors b as possible for which $Ax = b$ ha solution.
(d) In elimination on A, what multiple of the third row is subtracted to knock the fourth row?

2.12 If A is an n by $n - 1$ matrix, and its rank is $n - 2$, what is the dimension o nullspace?

2.13 Use elimination to find the triangular factors in $A = LU$, if

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

Under what conditions on the numbers a, b, c, d are the columns linearly indepenc

2.14 Do the vectors (1, 1, 3), (2, 3, 6), and (1, 4, 3) form a basis for $\mathbf{R}^3$?

2.15 Give examples of matrices A for which the number of solutions to $Ax = b$ is
(i) 0 or 1, depending on b;
(ii) ∞, independent of b;
(iii) 0 or ∞, depending on b;
(iv) 1, regardless of b.

2.16 In the previous exercise, how is r related to m and n in each example?

2.17 If x is a vector in $\mathbf{R}^n$, and $x^{\mathrm{T}}y = 0$ for every y, prove that $x = 0$.

2.18 If A is an n by n matrix such that $A^2 = A$ and rank $A = n$, prove that $A = I$.

2.19 What subspace of 3 by 3 matrices is spanned by the elementary matrices E_{ij}, ones on the diagonal and at most one nonzero entry below?

2.20 How many 5 by 5 permutation matrices are there? Are they linearly indepen Do they span the space of all 5 by 5 matrices? No need to write them all dow

2.21 What is the rank of the n by n matrix with every entry equal to one? How abo "checkerboard matrix," with $a_{ij} = 0$ when $i + j$ is even, $a_{ij} = 1$ when $i + j$ is o

2.22 (a) $Ax = b$ has a solution under what conditions on b, if

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}?$$

(b) Find a basis for the nullspace of A.
(c) Find the general solution to $Ax = b$, when a solution exists.
(d) Find a basis for the column space of A.
(e) What is the rank of A^T?

2.23 How can you construct a matrix which transforms the coordinate vectors e_1, e_2, e_3 into three given vectors v_1, v_2, v_3? When will that matrix be invertible?

2.24 If e_1, e_2, e_3 are in the column space of a 3 by 5 matrix, does it have a left-inverse? Does it have a right-inverse?

2.25 Suppose T is the linear transformation on $\mathbf{R}^3$ that takes each point (u, v, w) to $(u + v + w, u + v, u)$. Describe what T^{-1} does to the point (x, y, z).

2.26 *True or false*: (a) Every subspace of $\mathbf{R}^4$ is the nullspace of some matrix.
(b) If A has the same nullspace as A^T, the matrix must be square.
(c) The transformation that takes x to $mx + b$ is linear (from $\mathbf{R}^1$ to $\mathbf{R}^1$).

2.27 Find bases for the four fundamental subspaces of

$$A_1 = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} [1 \quad 4].$$

2.28 (a) If the rows of A are linearly independent (A is m by n) then the rank is ______ and the column space is ______ and the left nullspace is ______.
(b) If A is 8 by 10 with a 2-dimensional nullspace, show that $Ax = b$ can be solved for every b.

2.29 Describe the linear transformations of the x–y plane that are represented with standard basis $(1, 0)$ and $(0, 1)$ by the matrices

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

2.30 (a) If A is square, show that the nullspace of A^2 contains the nullspace of A.
(b) Show also that the column space of A^2 is contained in the column space of A.

2.31 When does the rank-one matrix $A = uv^T$ have $A^2 = 0$?

2.32 (a) Find a basis for the space of all vectors in $\mathbf{R}^6$ with $x_1 + x_2 = x_3 + x_4 = x_5 + x_6$.
(b) Find a matrix with that subspace as its nullspace.
(c) Find a matrix with that subspace as its column space.

2.33 Suppose the matrices in $PA = LU$ are

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & -3 & 2 \\ 2 & -1 & 4 & 2 & 1 \\ 4 & -2 & 9 & 1 & 4 \\ 2 & -1 & 5 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 4 & 2 & 1 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) What is the rank of A?
(b) What is a basis for the row space of A?
(c) *True or false*: Rows 1, 2, 3 of A are linearly independent.
(d) What is a basis for the column space of A?
(e) What is the dimension of the left nullspace of A?
(f) What is the general solution to $Ax = 0$?

2.5.16 (a) $-y_1 - y_4 - y_3 = 0,\ y_1 - y_2 = 0,\ y_2 + y_3 - y_5 = 0$ (b) By adding the 3 equations. (c) 3 (d) They correspond to the two independent loops $y_1y_2y_3$ and $y_3y_5y_4$.

2.6.4 Ellipse. **2.6.10** e^t, e^{-t}. **2.6.11** $\frac{x+y}{2}e^t + \frac{x-y}{2}e^{-t}$; $\begin{bmatrix} \frac12 & \frac12 \\ \frac12 & -\frac12 \end{bmatrix}$.

2.6.13 Suppose $u = A^{-1}x$ and $v = A^{-1}y$. Since A is linear, $A(cu + dv) = cAu + dAv = cx + dy$. Therefore $A^{-1}(cx + dy) = cu + dv$ and A^{-1} is linear. Since the transformations satisfy $A^{-1}A = I$, the product rule means that the corresponding matrices satisfy $M^{-1}M = I$.

2.6.16 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$; the double transpose of a matrix gives the matrix itself.

2.6.19 $p(x), q(x) \in S \Rightarrow \int_0^1 (cp(x) + dq(x))\,dx = c\int_0^1 p(x)\,dx + d\int_0^1 q(x)\,dx = 0 \Rightarrow cp(x) + dq(x) \in S \Rightarrow S$ is a subspace; $-\frac12 + x,\ -\frac13 + x^2,\ -\frac14 + x^3$ is a basis for S.

CHAPTER 3

3.1.2 (1, 1) and (1, 2); (1, 1) and (0, 0).

3.1.3 $(x_2/x_1)(y_2/y_1) = -1 \Rightarrow x_1y_1 + x_2y_2 = 0 \Rightarrow x^Ty = 0$.

3.1.6 All multiples of $(1, 1, -2)$; $(1/\sqrt3, 1/\sqrt3, 1/\sqrt3)$, $(1/\sqrt2, -1/\sqrt2, 0)$, $(1/\sqrt6, 1/\sqrt6, -2/\sqrt6)$.

3.1.8 $x \in V, x \in W \Rightarrow x^Tx = 0 \Rightarrow x = 0$. **3.1.10** $x_1 + x_2 - x_3 = 0$.

3.1.11 $A^Ty = 0 \Rightarrow y^Tb = y^TAx = (y^TA)x = 0$, which contradicts $y^Tb \neq 0$.

3.1.14 $(x - y)^T(x + y) = 0 \Leftrightarrow x^Tx + x^Ty - y^Tx - y^Ty = 0 \Leftrightarrow x^Tx = y^Ty \Leftrightarrow \|x\| = \|y\|$.

3.1.18 $\mathbf{R}^4$; the orthogonal complement is spanned by (1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0).

3.1.20 It means that every vector which is orthogonal to all vectors orthogonal to S is in S.

3.1.22 One basis is (1, 1, 1, 1).

3.2.1 (a) $(x + y)/2 \geq \sqrt{xy}$. (b) $\|x + y\|^2 \leq (\|x\| + \|y\|)^2 \Rightarrow (x + y)^T(x + y) \leq \|x\|^2 + 2\|x\|\,\|y\| + \|y\|^2 \Rightarrow x^Ty \leq \|x\|\,\|y\|$.

3.2.3 (10/3, 10/3, 10/3); (5/9, 10/9, 10/9). **3.2.5** $\arccos(1/\sqrt{n})$; $\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ $[1/n \cdots 1/n]$.

3.2.7 $b = (1, \ldots, 1)$; $a_1 = \cdots = a_n$. **3.2.9** $P^2 = \frac{aa^Taa^T}{a^Taa^Ta} = \frac{a(a^Ta)a^T}{a^Taa^Ta} = \frac{aa^T}{a^Ta} = P$.

3.2.11 (a) $\begin{bmatrix} \frac{1}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{9}{10} \end{bmatrix}$; $\begin{bmatrix} \frac{9}{10} & -\frac{3}{10} \\ -\frac{3}{10} & \frac{1}{10} \end{bmatrix}$. (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. The sum of the projections onto two perpendicular lines gives the vector itself. The projection onto one line and then another (which is perpendicular to the first one) always gives {0}.

3.2.13 $\frac{a_1a_1}{a^Ta} + \cdots + \frac{a_na_n}{a^Ta} = 1$. **3.2.14** $\begin{bmatrix} \frac16 & -\frac13 & \frac16 \\ -\frac13 & \frac23 & -\frac13 \\ \frac16 & -\frac13 & \frac16 \end{bmatrix}$.

3.2.16 (a) $P^T = P \Rightarrow (Px)^Ty = x^TP^Ty = x^T(Py)$. (b) No; $1/\sqrt{15}$, $1/3\sqrt3$.
(c) $P^2 = P \Rightarrow (Px)^TPy = x^TP^TPy = x^TP^2y = x^T(Py)$, 0° or 180°

3.3.1 2; $(10 - 3x)^2 + (5 - 4x)^2$; $(4, -3)(3, 4)^T = 0$.

3.3.3 $\bar{x} = \begin{bmatrix} \frac13 \\ \frac13 \end{bmatrix}$; $p = \begin{bmatrix} \frac13 \\ \frac13 \\ \frac23 \end{bmatrix}$; $b - p = \begin{bmatrix} \frac23 \\ \frac23 \\ -\frac23 \end{bmatrix}$.

3.3.5 $6 + (5/2)t$; (7/2, 6, 17/2). **3.3.8** column space S; rank k.

3.3.10 $A^TA = I$, $A^Tb = 0$; 0.

3.3.12 (a) (−1, 1, 0, 0), (−1, 0, 0, 1) (b) $\begin{bmatrix} \frac13 & \frac13 & 0 & \frac13 \\ \frac13 & \frac13 & 0 & \frac13 \\ 0 & 0 & 1 & 0 \\ \frac13 & \frac13 & 0 & \frac13 \end{bmatrix}$ (c) (0, 0, 0, 0)

3.3.13 $61/35 - (36/35)t$; (133/35, 95/35, 61/35, −11/35).

3.3.15 $H^2 = (I - 2P)^2 = I - 4P + 4P^2 = I - 4P + 4P = I$.

3.3.18 (1) $C + D + E = 3$, $C + 3E = 6$, $C + 2D + E = 5$, $C = 0$ (No
(2) $\begin{bmatrix} 4 & 3 & 5 \\ 3 & 5 & 3 \\ 5 & 3 & 11 \end{bmatrix}\begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 26 \end{bmatrix}$.

3.3.19 $A^T(AA^T)^{-1}A$. **3.3.21** $\begin{bmatrix} a_1^Ta_1 & -a_1^Ta_2 \\ -a_1^Ta_2 & a_2^Ta_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_1^T \\ -a_2^T \end{bmatrix}$

3.3.24 $-3/10 - (12/5)t$. **3.3.25** $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$; $\begin{bmatrix} C \\ D \\ E \end{bmatrix}$; $\begin{bmatrix} 2 \\ 0 \\ -3 \\ -5 \end{bmatrix}$

3.4.1 (a) $-4 = C - 2D$, $-3 = C - D$, $-1 = C + D$, $0 = C + 2D$
(c) b is in the column space; b itself.

3.4.3 (−2/3, 1/3, −2/3); the sum is b itself; notice that $a_1a_1^T$, a_2a onto three orthogonal directions. Their sum is projection on should be the identity.

3.4.5 $(I - 2uu^T)^T(I - 2uu^T) = I - 4uu^T + 4uu^Tuu^T = I$; $Q = \begin{bmatrix} \frac12 \\ -\frac12 \\ \frac12 \\ \frac12 \end{bmatrix}$

3.4.7 $(x_1q_1 + \cdots + x_nq_n)^T(x_1q_1 + \cdots + x_nq_n) = x_1^2 + \cdots + x_n^2 \Rightarrow x_1^2 + \cdots + x_n^2$.

3.4.9 $0q_1 + 0q_2$. **3.4.12** 2; $\begin{bmatrix} 1/\sqrt2 & 1/\sqrt2 \\ 1/\sqrt2 & -1/\sqrt2 \end{bmatrix}\begin{bmatrix} \sqrt2 & 2\sqrt2 \\ 0 & 2\sqrt2 \end{bmatrix}$

3.4.13 $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.