1. *Recall the complex roots of 1.*
a) Show that the n complex solutions of the equation $z^n = 1$, called *the* n^{th} *roots of unity*, are $z_k = \exp[2\pi ik/n],\ k = 0, 1, \ldots, n-1$.
b) z_1 is called the *primitive* n^{th} *root of unity*. Show that $z_k = z_1^k$.
c) Why do we stop the index at $k = n-1$? What is z_n, z_{n+1}?
d) Plot these solutions $z_0,\ z_1, \ldots, z_{n-1}$ in the the complex plane for $n = 2$, then, on a separate plane, for $n = 3$, then for $n = 4$ and then for general n.

Recall the formula $z^n - 1 = (z-1)(z^{n-1} + z^{n-2} + \ldots + z + 1)$. This shows that $z_1, z_2, \ldots, z_{n-1}$ satisfy $z^{n-1} + z^{n-2} + \ldots + z + 1 = 0$.

2. Let $w = e^{2\pi i/n}$ be the primitive n^{th} root of unity.
Consider the matrix

$$U = \frac{1}{\sqrt{n}}[w^{jk}]_{j,k=0,1,\ldots,n-1}$$

which is, in fact, the Fourier Matrix:

$$U = \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{n-1} \\ 1 & w^2 & w^4 & \cdots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \ldots & w^{(n-1)^2} \end{bmatrix}$$

Show that U is unitary.

3. *Fundamental spaces for products of matrices* AB. Show that:
a) The null space of AB contains the null space of B:
$\mathcal{N}(AB) \supset \mathcal{N}(B)$
b) The column space of AB is contained in the column space of A:
$\mathcal{R}(AB) \subset \mathcal{R}(A)$
c) The left null space of AB contains the left null space of A:
$\mathcal{N}((AB)^*) \supset \mathcal{N}(A^*)$
d) The row space of AB is contained in the space of B:
$\mathcal{R}((AB)^*) \subset \mathcal{R}(B^*)$

4. Find the matrix P which takes any vector in $\mathbb{R}^4$ to its orthogonal projection on the (2-d) plane containing the vectors $(1, 4, 4, 1)$ and $(2, 9, 8, 2)$.