Math 5101 (AU 2013) **Homework 5** Name(s):

1. For each of the following matrices

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

a) Find the null space.
b) Find the column space.
c) Is the matrix invertible? If so, find its inverse, and check your answer.
d) Does the matrix have a left inverse? If so, find it, and check your answer.
e) Does the matrix have a right inverse? If so, find it, and check your answer.

2. Construct a matrix which transforms the standard basis vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ of $\mathbb{R}^3$ into three given vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{R}^3$. When is this matrix invertible?

3. Let M be an invertible matrix. Assume M has n independent eigenvectors $\mathbf{v}_1, \dots, \mathbf{v}_n$, corresponding to the eigenvalues $\lambda_1, \dots, \lambda_n$.

a) If $\mathbf{v}$ is an eigenvector of M, show that $\mathbf{v}$ is also an eigenvector of M^2 and M^{-2}. What is the corresponding eigenvalue?

b) Find the eigenvectors and eigenvalues of M^k for any integer k.

c) Find the eigenvectors and eigenvalues of $M + cI$ (where c is a scalar and I is the identity matrix).

d) Find the eigenvectors and eigenvalues of $M^2 - 10M + 25I$.

4. For each of the matrices A, B below: find the eigenvalues and eigenvectors.

If the matrix is diagonalizable explain why this is the case, find its diagonal form and a transition matrix. [Otherwise, find a Jordan normal form and a transition matrix –**if we get there on Friday**].

$$A = \begin{bmatrix} 5 & 4 & 2 & -4 \\ 0 & 1 & 0 & 0 \\ -2 & -2 & 0 & 2 \\ 2 & 2 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 5 & 2 & -4 \\ 0 & 1 & 0 & 0 \\ -2 & -2 & 0 & 2 \\ 2 & 3 & 1 & -1 \end{bmatrix}$$