Math 5101 HOMEWORK 3

1. Are the functions $e^t, t, \ln t$ linearly independent? Explain.

2. Determine the nature of the equilibrium at zero of the system $\dot{\mathbf{x}} = M\mathbf{x}$ where

$$M = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} \tag{1}$$

and draw the phase portrait.

Same question for

$$M = \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix} \tag{2}$$

3. Find the general solution of the recurrence

$$a_{n+1} = 3a_n - a_{n-1} \tag{3}$$

and the particular solution when $a_0 = a_1 = 1$. Find a z s.t. $\lim_{n\to\infty} a_n z^n$ is a constant, and find the constant.

Find the general solution of the recurrence

$$a_{n+1} = 2a_n - a_{n-1} \tag{4}$$

and the particular solution when $a_0 = a_1 = 1$.

Bonus problem [by Fibonacci] (+20p) **You are not required to solve it**.

Three men owned some money, their shares being 1/2, 1/3, and 1/6. Each took some money at random until none was left. The first man then returned 1/2 of what he had taken, the second 1/3 and the third 1/6. When the money now in the pile was divided equally among the men, each possessed what he was entitled to. How much money was in the original store, and how much did each man take?