1. Are the matrices

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 2 & -2 \\ 2 & 1 & -1 \end{bmatrix}; \; B = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 4 \\ -2 & 0 & 3 \end{bmatrix}$$

simultaneously diagonalizable? If so, find a basis in which they are both diagonal.

2. Show that any two-by-two matrix R with the property $R^2 = I$ is of one of the following forms.

$$\begin{bmatrix} \alpha & \beta \\ \frac{-\alpha^2+1}{\beta} & -\alpha \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

or the transpose of the first matrix, where α is arbitrary, $\beta \neq 0$.

3. Let M be a matrix with distinct positive eigenvalues and let C be such that $C^2 = M$. Show that M and C commute. (Be careful: if you want to invert something, you have to show invertibility.)

4. Show that there are 2×2 matrices with real coefficients s.t. $R^2 = -I$.

Bonus (+10p) You are not required to solve it .

Let M be a matrix with distinct positive eigenvalues. Find all the matrices with the property $R^2 = M$–the answer should be expressed in terms of the eigenvectors and eigenvalues of M. How many such matrices are there?