Name :__________________________

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Math 5102, 2015

Take Home Final

You can use books, notebooks, web resources, electronic computational devices/software.

You are not to receive any help from anyone else on this exam, nor are you to give any help to anyone else on this exam.

Evidence of any collaboration will result in negative points.

By signing your name below, you are acknowledging that you understand and accept these rules.

Signature:__________________________

Problem	Possible	Received
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1. Solve the heat equation in $\mathbb{R}^3$

$$u_t = \Delta u; \quad u(t=0; x, y, z) = \cos(x)\cos(y)\cos(z)$$

Justify your answer!

2. Find the best approximation, in square average, of the function $\cos x$, for $x \in [0, 3\pi/4]$, by polynomials of degree at most 2.

3. Consider the equation $(1-x^2)y'' - 2xy' + \lambda y = 0$ where λ is a parameter.

a) Bring the equation to a self-adjoint form for $x \in [-1, 1]$.

b) Show that this is a self-adjoint problem on $D = \{y \in L^2[-1,1] \,|\, y', y'' \in L^2[-1,1]\}$

c) Show that all eigenvalues are nonnegative.

4. Consider further the equation in problem **3**, $(1-x^2)y'' - 2xy' + \lambda y = 0$.

a) Show that the functions $p_0(x) = 1$ and $p_1(x) = x$ are eigenfunctions, and find the corresponding eigenvalues $\lambda = \lambda_0$, respectively $\lambda = \lambda_1$.

b) It can be shown that the eigenvalues are $\lambda_n = n(n+1)$ $(n = 0, 1, \ldots)$, all simple, and that the eigenfunctions are polynomials, $p_n(x)$. Explain why functions $f(x)$ *continuous* on $[-1, 1]$ can be expanded as a series $f = \sum_{n=1}^{\infty} c_n p_n$ for some numbers c_n. Explain in which sense the series is convergent and write formulas for calculating c_n in terms of $f(x)$ and $p_0, p_1, p_2, \ldots$.

5. Give a *simple* example of a Sturm-Liouville problem whose lowest eigenvalue is negative.